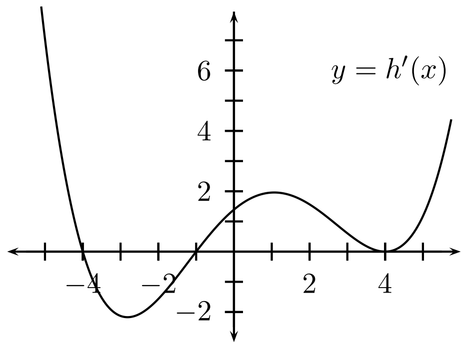
1. [13 points] The graph of the **derivative**, $h'(x)$, of a continuous function h is shown below:



a. [3 points] Approximate the x-coordinates of all critical points of h in the interval $(-5, 5)$, and classify each as either a local maximum, a local minimum, or neither.

Solution: Since $h'(x)$ is defined at every point in $(-5, 5)$, the critical points of h on $(-5, 5)$ are the zeros of h'. These are $x = -4$, $x = -1$, and $x = 4$. At $x = -4$, h' changes from positive to negative, so h has a *local maximum* at $x = -4$. At $x = -1$, h' changes from negative to positive, so h has a *local minimum* at $x = -1$. Since the sign of h' does not change at $x = 4$, h has *neither a local maximum nor local minimum* at $x = 4$.

b. [3 points] Approximate the x-coordinate(s) of any inflection point(s) of h in the interval $(-5, 5)$.

Solution: At (approximately) $x = -2.8$, $x = 1$, and $x = 4$, the sign of the derivative of h' changes, so the concavity of h changes. Hence h has an inflection point at each of $x = -2.8$, $x = 1$, and $x = 4$.

c. [2 points] Approximate the value(s) of x on the interval $[-5, 5]$ where h attains its global maximum.

Solution: h attains its global maximum on $[-5, 5]$ at $x = 5$.
(By comparing areas, we see that the amount by which h decreases between $x = -4$ and $x = -1$ is less than the amount by which it increases after $x = -1$.)

d. [2 points] Approximate the value(s) of x on the interval $[-5, 5]$ where h attains its global minimum.

Solution: h attains its global minimum on $[-5, 5]$ at $x = -1$.
(The area under the graph of h' between -5 and -4 is less than the area above the graph of h' between $x = -4$ and $x = -1$, so $h(-1) < h(-5)$. Then h increases after $x = -1$.)

e. [3 points] If $h(1) = 3$, find the best linear approximation to $h(x)$ at the point $x = 1$. Is this linear approximation an underestimate or an overestimate of h for points near $x = 1$? Explain.

Solution: The best linear approximation to $h(x)$ at the point $x = 1$ is given by $L(x) = h(1) + h'(1)(x - 1)$. So since $h(1) = 3$ and $h'(1) = 2$, we have $L(x) = 3 + 2(x - 1)$.
This linear approximation is an underestimate of $h(x)$ for nearby $x < 1$ (since h is concave up to the left of $x = 1$). Similarly, $L(x)$ is an overestimate of $h(x)$ for nearby $x > 1$ (since h is concave down to the right of $x = 1$).