3. [12 points]
Use the information in the table below to answer (a) - (c):

x	-3	-2	-1	0	1	2	3
$f(x)$	1	3	2	-1	-3	-2	0
$f'(x)$	3	2	-2	-3	-1	1	2
$g(x)$	2	3	1	2	4	3	1
$g'(x)$	1	-2	2	3	-1	0	-3

a. [4 points] If $h(x) = g(f(e^\pi \ln x))$, find $h'(1)$. (Give an exact answer.)

Solution: By the Chain Rule, we have $h'(x) = g'(f(e^\pi \ln x))f'(e^\pi \ln x)e^\pi \left(\frac{1}{x}\right)$. So,

$$\begin{aligned} h'(1) &= g'(f(e^\pi \ln 1))f'(e^\pi \ln 1)e^\pi \left(\frac{1}{1}\right) = g'(f(0))f'(0)e^\pi(1) \\ &= g'(-1)(-3)(e^\pi)(1) = 2(-3)(e^\pi)(1) = -6e^\pi. \end{aligned}$$

b. [4 points] If $j(x) = \sin^2\left(\frac{3f(x)}{2}\right)$, find $j'(-2)$. (Give an exact answer.)

Solution: By the Chain Rule, we have $j'(x) = 2\sin\left(\frac{3f(x)}{2}\right)\cos\left(\frac{3f(x)}{2}\right)\frac{3}{2}f'(x)$. Thus

$$\begin{aligned} j'(-2) &= 2\sin\left(\frac{3f(-2)}{2}\right)\cos\left(\frac{3f(-2)}{2}\right)\frac{3}{2}f'(-2) \\ &= 2\sin\left(\frac{9}{2}\right)\cos\left(\frac{9}{2}\right)\frac{3}{2}(2) = 6\sin\left(\frac{9}{2}\right)\cos\left(\frac{9}{2}\right) (= 3\sin 9). \end{aligned}$$

c. [4 points] Give an exact answer for $\displaystyle\int_{-3}^{2} \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}\,dx$, assuming $g(x) \neq 0$.

Solution: By the Fundamental Theorem of Calculus and Quotient Rule,

$$\int_{-3}^{2} \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}\,dx = \frac{f(2)}{g(2)} - \frac{f(-3)}{g(-3)} = \frac{-2}{3} - \frac{1}{2} = -\frac{7}{6}.$$