4. [12 points]

a. [5 points] If the average value of a continuous function g on $[1, 8]$ is 3, find

$$\int_{-1}^{6} 3\,(g(x+2)) + 5\,dx.$$

Solution: We are given that

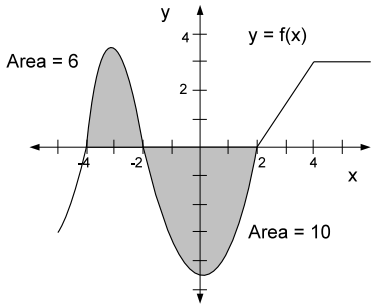
$$\frac{1}{8-1}\int_1^8 g(x)\,dx = 3, \text{ so, } \int_1^8 g(x)\,dx = 21.$$

$$\text{Thus, } \int_{-1}^{6} 3\,(g(x+2)) + 5\,dx = 3\int_{-1}^{6} g(x+2)\,dx + \int_{-1}^{6} 5\,dx,$$

which gives

$$3\int_1^8 g(x)\,dx + 5(6-(-1)) = 3(21) + 35 = 98.$$

Use the following graph of a function $f(x)$ to compute the quantities in parts (b)–(d) below.



b. [2 points] $\displaystyle\int_{-4}^{2} f(x)\,dx$

Solution: $\displaystyle\int_{-4}^{2} f(x)\,dx = 6 - 10 = -4.$

c. [3 points] The area between the graph of $f(x)$ and the x–axis for $-4 \le x \le 5$ if the units on the axes are centimeters.

Solution: Area $= 6 + 10 + 3 + 3 = 22$ cm^2.

d. [2 points] $\displaystyle\int_{3}^{5} f'(x)\,dx$

Solution: $\displaystyle\int_{3}^{5} f'(x)\,dx = f(5) - f(3) = 3 - \frac{3}{2} = \frac{3}{2}.$