

6. [12 points] The rate $q(t)$ at which cars passed through the intersection of Main Street and Huron after a football game is presented in the table below.

t (in minutes after the game ended)	0	20	40	60	80	100	120
$q(t)$ (in cars per minute)	10	15	19	21	20	17	13

- a. [4 points] What is the meaning of $\int_0^{120} q(t) dt$? Using a left Riemann sum and $n = 6$, estimate $\int_0^{120} q(t) dt$. (Write out the terms of your sum.)

Solution: The expression $\int_0^{120} q(t) dt$ gives the total number of cars that passed through the intersection in the first two hours after the game. A left-hand approximation with 6 subdivisions is given by

$$(20)(10 + 15 + 19 + 21 + 20 + 17) = 2040 \text{ cars .}$$

- b. [2 points] Write an expression for the average rate at which cars passed through the intersection for the first two hours after the game ended.

Solution: The average rate at which cars passed through the intersection during this time period is given by $\frac{1}{120} \int_0^{120} q(t) dt$.

- c. [3 points] Estimate $q'(30)$.

Solution: The best estimate we can get from the table is

$$q'(30) \approx \frac{19 - 15}{40 - 20} = 0.2 \text{ cars per minute per minute.}$$

- d. [3 points] If $Q(t)$ denotes the total number of cars that have passed through the intersection t minutes after the game ended, find and interpret $Q'(60)$.

Solution: We can read $Q'(60)$ from the table. We have $Q'(60) = 21$ and indicates that one hour after the game, approximately 21 additional cars would pass through the intersection in the next minute.