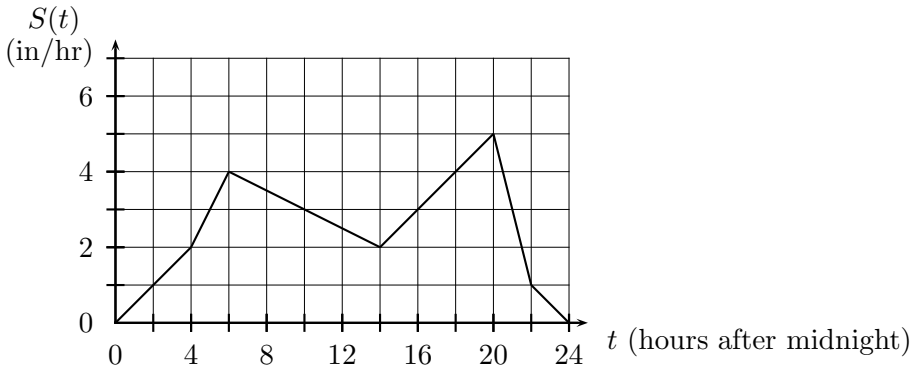
8. [12 points]
The graph below gives the rate $S(t)$, in inches per hour, of snow fall t hours after midnight along a major thoroughfare in Ann Arbor. Beginning at 8:00 a.m., the city truck began removing snow at the rate of 2 in/hr. [Salting had been halted, as a consequence of economic conditions in Michigan.] Assume that there was no snow on the road prior to midnight.



a. [2 points] How deep was the snow at 2:00 p.m.?

Solution: Since the snow removal had been going on for 6 hours at 2 pm, the depth of the snow at 2:00 pm was $\int_0^{14} S(t)\,dt - 2(6) = 34 - 12 = 22$ inches. (Yes, it was a serious storm.)

b. [2 points] At what time was the snow falling the fastest?

Solution: The snow was falling fastest at $t = 20$ which is 8 pm.

c. [2 points] At what time was the snow deepest?

Solution: Since the snow removal began at 8 am, the rate at which the depth was changing is given by $S(t)$ for $0 \le t < 8$ and then by $S(t) - 2$ for $t \ge 8$. This rate is positive until $t = 21.5$ (when the line $y = 2$ intersects the graph of $S(t)$ for the second time), which is 9:30 pm. So the snow is deepest at 9:30 pm,

d. [2 points] At what time was the depth of the snow on the ground increasing fastest?

Solution: As in part (c), above, the rate at which the depth of snow is changing is given by $S(t)$ for $0 \le t < 8$ and then by $S(t) - 2$ for $t \ge 8$. We can see from the graph that this rate is greatest at time $t = 6$, i.e. at 6:00 am.

e. [2 points] What is the average rate at which snow fell between 4 am and 2 pm?

Solution: The average rate at which snow fell between 4 am and 2 pm is given by $\frac{1}{14-4}\int_4^{14} S(t)\,dt = \frac{1}{10}(30) = 3$ inches per hour.

f. [2 points] Write an expression for the average depth of the snow on the ground between 5 am and 8 am.