8. [15 points] In each part of this problem, write "True" on the blank line for all statements which *must* be true based on the information given. If the statement is not necessarily true, write "False."

a. [5 points] The function $g(x)$ is differentiable on $(-\infty, \infty)$ and $g'(3) = 0$.

- _____ The function $g(x)$ is continuous for all real values of x.

- _____ The function $g(x)$ has a local maximum or a local minimum at $x = 3$.

- _____ The second derivative of g exists at $x = 3$.

- _____ The derivative of $(x \cdot g(x))^2$ at $x = 3$ is equal to 0.

- _____ The derivative of $g(x)$ at $x = 2$ exists.

b. [5 points] A differentiable function $v(t)$ gives the velocity of a particle at a time $t \geq 0$. The function v is positive for all t in its domain.

- _____ The integral $\int_a^b v(t)dt$ is the total distance traveled by the particle between $t = a$ and $t = b$ for $0 \leq a \leq b$.

- _____ The function $v'(t)$ gives the acceleration of the particle at a time $t \geq 0$.

- _____ The function $v'(t)$ is positive for some value of t.

- _____ The average velocity of the particle between $t = 1$ and $t = 2$ is $\int_1^2 v(t)dt$.

- _____ The particle is traveling in the same direction at all times.

c. [5 points] Let $g(R)$ be the amount of natural gas in liters used by an R rated furnace in an hour of operation. The rating of a furnace is a number between 0 and 100 which is related to the efficiency of the furnace. The higher the rating of a furnace, the more efficient it is. Suppose $g'(95) = -0.01$, $(g^{-1})'(2) = -40$, $g(95) = 1$, and $g^{-1}(2) = 40$.

- _____ It is reasonable to expect that a furnace which uses one liter of natural gas in an hour has a rating which is approximately 40 more than a furnace which uses two liters in an hour.

- _____ It is reasonable to expect that a furnace which uses 1.9 liters of natural gas in an hour has a rating which is approximately 4 more than a furnace which uses two liters in an hour.

- _____ It is reasonable to expect that in one hour of operation, a furnace with a rating of 90 uses about 0.05 more liters of natural gas than a furnace with a rating of 95.

- _____ For each one point rating drop from a rating of 95, a furnace will use 0.01 more liters of natural gas in one hour of operation.

- _____ A furnace with a rating of 40 uses two liters of natural gas in an hour of operation.