2. [13 points] The U-value of a wall of a building is a positive number related to the rate of energy transfer through the wall. Walls with a lower U-value keep more heat in during the winter than ones with a higher U-value. Consider a wall which consists of two materials, material A with U-value a and material B with U-value b. The U-value of the wall w is given by

$$w = \frac{ab}{b+a}.$$

Considering a as a constant, we can think of w as a function of b, $w = u(b)$.

a. [4 points] Write the limit definition of the derivative of $u(b)$.

Solution: The derivative of $u(b)$ is defined to be

$$u'(b) = \lim_{h\to 0} \frac{u(b+h) - u(b)}{h} = \lim_{h\to 0} \frac{a(b+h)/(b+h+a) - ab/(b+a)}{h}$$

b. [4 points] Calculate $u'(b)$. (You do **not** need to use the limit definition of the derivative for your calculation.)

Solution: By the quotient rule,

$$u'(b) = \frac{(b+a)(a) - ab}{(b+a)^2} = \frac{a^2}{(b+a)^2}.$$

c. [5 points] Find the x- and y-coordinates of the global minimum and maximum of $u(b)$ for b in the interval $[1, 2]$. Your answer may involve the parameter a. [Recall that $a, b > 0$.]

Solution: The derivative $u'(b)$ is strictly positive for all $b > 0$ by part **b.** This means there are not any critical points on $[1, 2]$ and u is strictly increasing so $b = 1$ is the global minimum while $b = 2$ is the global maximum. Now we compute $u(1) = \frac{a}{1+a}$ and $u(2) = \frac{2a}{2+a}$.

Global minimum on $[1, 2]$: $(1, \frac{a}{1+a})$

Global maximum on $[1, 2]$: $(2, \frac{a}{2+a})$