5. [12 points] A certain type of spherical melon has weight proportional to its volume as it grows. When the melon weighs 0.2 pounds, it has a volume of 36 cm^3 and its weight is increasing at a rate of 0.1 pounds per day. [Note: The volume of a sphere is $V = \frac{4}{3}\pi r^3$.]

a. [3 points] Find $\dfrac{dV}{dt}$ when the melon weighs 0.2 pounds (t measured in days).

Solution: When the melon weighs 0.2 pounds, it has a volume of 36 cm^3, so if $V = kw$ where V is volume, w is weight and k is a proportionality constant, then $36 = 0.2k$ giving $k = 180$. Now $\frac{dV}{dt} = k\frac{dw}{dt} = 180(0.1) = 18$ cm^3 per day.

b. [5 points] Find the rate at which the radius of the melon is increasing when it weighs 0.2 pounds.

Solution: We are looking for $\frac{dr}{dt}$ when $w = 0.2$. Differentiating the equation for a sphere gives

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

We know $36 = \frac{4}{3}\pi r^3$, so $r = \frac{3}{\pi^{1/3}}$. We also know that $\frac{dV}{dt} = 18$. Putting this together we have

$$18 = 4\pi \frac{9}{\pi^{2/3}} \frac{dr}{dt}.$$

which means $\frac{dr}{dt} = \frac{1}{2\pi^{1/3}}$ cm per day.

c. [4 points] Use a local linearization to approximate the volume of the melon 36 hours after it weighs 0.2 pounds.

Solution: Since $\frac{dV}{dt} = 18$ cm^3 per day and 36 hours is 1.5 days we know the melon will increase by approximately 18(1.5)=27 cm^3 in the 36 hours after it weighs 0.2 pounds. Since it is 36 cm^3 at the time in question, the volume will be about 63 cm^3 36 hours later.