

1. [14 points] You are online playing the Facebook-based game, FarmVille, and you receive land with 5 stalks of corn on it. You decide that you would like to model the corn population on this patch of land using your calculus skills, so you recall that a good model for population growth is the logistic model

$$P(t) = \frac{L}{1 + Ae^{-kt}} \quad L > 0, \quad A > 0, \quad k > 0.$$

- a. [5 points] Using the *limit definition of the derivative*, write an explicit expression for the derivative of the function $P(t)$ at $t = 1$. Do not evaluate this expression.

- b. [5 points] Using the definition of the logistic model above, compute the following in terms of L , k , and A , showing your work or providing an explanation for each part:

i. [1 points] $\lim_{t \rightarrow \infty} P(t)$

ii. [1 points] $\lim_{t \rightarrow -\infty} P(t)$

iii. [1 points] $P(0)$

iv. [2 points] $P'(0)$

- c. [4 points] Your farmland satisfies the following conditions:

$$P(0) = 5, \quad P'(0) = 1, \quad \lim_{t \rightarrow \infty} P(t) = 100.$$

Based on your answers in part (b), compute the correct values of L , k , and A for the logistic equation modeling corn population on your land.

$$L = \underline{\hspace{2cm}} \quad A = \underline{\hspace{2cm}} \quad k = \underline{\hspace{2cm}}$$