1. [14 points] You are online playing the Facebook-based game, FarmVille, and you receive land with 5 stalks of corn on it. You decide that you would like to model the corn population on this patch of land using your calculus skills, so you recall that a good model for population growth is the logistic model

\[ P(t) = \frac{L}{1 + Ae^{-kt}} \quad L > 0, \quad A > 0, \quad k > 0. \]

a. [5 points] Using the limit definition of the derivative, write an explicit expression for the derivative of the function \( P(t) \) at \( t = 1 \). Do not evaluate this expression.

b. [5 points] Using the definition of the logistic model above, compute the following in terms of \( L, k, \) and \( A \), showing your work or providing an explanation for each part:

i. [1 points] \( \lim_{t \to \infty} P(t) \)

ii. [1 points] \( \lim_{t \to -\infty} P(t) \)

iii. [1 points] \( P(0) \)

iv. [2 points] \( P'(0) \)

c. [4 points] Your farmland satisfies the following conditions:

\[ P(0) = 5, \quad P'(0) = 1, \quad \lim_{t \to \infty} P(t) = 100. \]

Based on your answers in part (b), compute the correct values of \( L, k, \) and \( A \) for the logistic equation modeling corn population on your land.

\[ L = \quad \quad \quad \quad \quad \quad \quad \quad A = \quad \quad \quad \quad \quad \quad k = \quad \quad \quad \quad \quad \quad \]