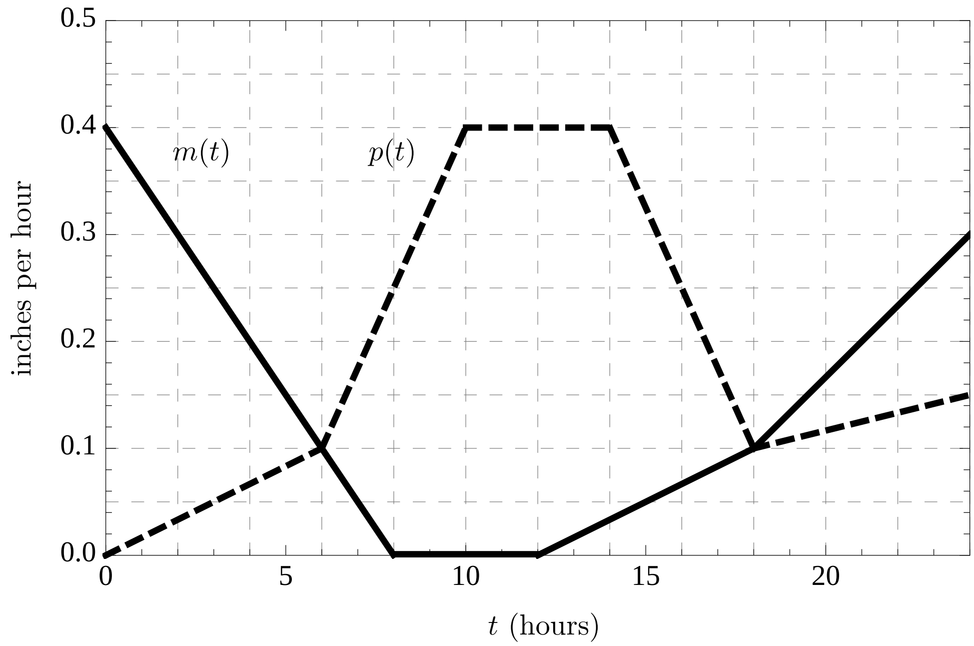
3. [16 points] Suppose the graph below shows the rate of snow melt and snowfall on Mount Arvon, the highest peak in Michigan (at a towering 1970 ft), during a day (24 hour period) in April of last year. The function $m(t)$ (the solid curve) is the rate of snow melt, in inches per hour, t hours after the beginning of the day. The function $p(t)$ (the dashed curve) is the snowfall rate in inches per hour t hours after beginning of the day. There were 18 inches of snow on the ground at the beginning of the day.



a. [2 points] Over what time period(s) was the snowfall rate greater than the snow melt rate?

Solution: The snowfall rate was greater than the snow melt rate between hours 6 and 18 when the snowfall (dotted) curve is above the snow melt (solid) curve.

b. [2 points] When was the amount of snow on Mount Arvon increasing the fastest? When was it decreasing the fastest?

Solution: The amount of snow was increasing the fastest between hours 10 and 12. The amount of snow was decreasing the fastest at the very beginning of the day ($t = 0$).

c. [3 points] When was the amount of snow on Mount Arvon the greatest? Explain.

Solution: The amount of snow was increasing between $t = 6$ and $t = 18$ and decreasing at all other times. This means there should be the most snow at $t = 18$ (when the amount of snow stopped increasing) or at $t = 0$ (before snow started melting). The area between the curves represents the increase ($p(t) > m(t)$) or decrease ($p(t) > m(t)$) in snow over a given period of time. By inspection of the graph, there was much more of an increase between $t = 6$ and $t = 18$ than there was a decrease between $t = 0$ and $t = 6$, so there must have been the most snow at the end of the 18th hour ($t = 18$).

d. [3 points] How much snow was there on Mount Arvon at the end of the day (at $t = 24$)? Show work.

Solution: If A is the area between $m(t)$ and $p(t)$ from $t = 0$ to $t = 6$, B is the between $m(t)$ and $p(t)$ from $t = 6$ to $t = 18$ and C is the area between $m(t)$ and $p(t)$ between $t = 18$ and $t = 24$. Each "box" counts for 0.1 inches of snow. The amount of snow at the end of the day will be

$$18 + 0.1(-A + B - C) = 18 + 0.1(-12 + 32 - 4.5) = 19.55 \text{ inches.}$$

3. (continued)

e. [6 points] The graph of $p(t)$ is repeated below. On the empty set of axes, sketch a well-labeled graph of $P(t)$, an antiderivative of $p(t)$ satisfying $P(0) = 0$. Label and give the coordinates of the points on the graph of $P(t)$ at $t = 10$ and $t = 18$.