

4. [10 points] Your family farm has a small herd of dairy cows. You decide to track the daily milk production levels of your favorite cow, Bessie, over the course of several months. Below is your table of measurements from every two months of Bessie's milk production level  $p(t)$  in liters per day:

day $t$	0	60	120	180	240	300	360
liters per day $p(t)$	18	24	18	6	0	6	18

- a. [4 points] The function modeling Bessie's daily milk production  $p(t)$  is sinusoidal. What is an equation for this function?

*Solution:* The period of this sinusoidal will be 360 (so  $B = \frac{\pi}{180}$ ), based on the information in the table with an amplitude of 12 and a vertical shift of 12 as well. Since the maximum is at 60 days instead of at 0, we will need to shift the cosine function to match the data. Therefore, an equation for this function is  $p(t) = 12 \cos\left(\frac{\pi}{180}(x - 60)\right) + 12$ . There are several other possible functions, including

$$p(t) = -12 \sin\left(\frac{\pi}{180}(t - 150)\right) + 12$$

- b. [3 points] Suppose you use a right-hand Riemann sum to approximate the total milk produced by Bessie between day 60 and day 240. Is this an overestimate, an underestimate, or can it not be determined? Explain your reasoning.

*Solution:* The key observation here is that between day 60 and day 240, the function  $p(t)$  is decreasing. The total milk produced by Bessie between day 60 and 240 is  $\int_{60}^{240} p(t) dt$ . When we use a right-hand Riemann sum to approximate the area under a decreasing curve, it gives an underestimate. Therefore, the right-hand sum will give an underestimate of the milk produced by Bessie.

- c. [3 points] How many measurements would you need to take between day 60 and day 240 to be sure your right-hand Riemann sum approximation is no more than 10 liters off from the exact amount of milk Bessie produces?

*Solution:* In order to figure out the error in the approximate of an integral by a Riemann sum, we use the formula  $|LHS - RHS| = |f(a) - f(b)|\frac{b-a}{n}$ . In this problem, we are only concerned with the interval (60, 240), we want to find the value of  $n$  which makes this difference less than or equal to 10. If we use the table, we can plug in all the necessary values and get

$$\left| (24 - 0) \frac{240 - 60}{n} \right| \leq 10 \Rightarrow 24(180) \leq 10n \Rightarrow n \geq 432.$$

It will take at least 432 measurements between day 60 and day 240 in order to approximate the exact amount of milk Bessie produces over that period.