5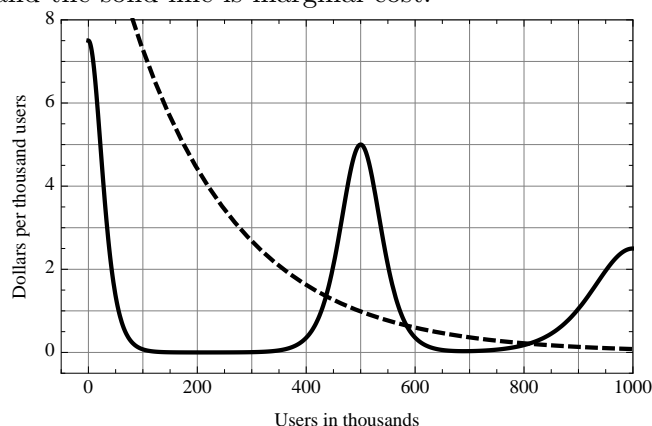. [8 points] The owners of a social networking site are concerned about their profit margins, so they develop the graphs for marginal revenue and marginal cost in terms of the number of users on the site. Use the graphs below to answer the following questions. The dashed graph is marginal revenue and the solid line is marginal cost.

a. [3 points] At which approximate number(s) of users is marginal cost equal to marginal revenue?

Solution: At around 450, 575, and 825 thousand users, the two curves intersect, so marginal cost is equal to marginal revenue at those three points.

b. [5 points] Which of your answers from part (a) maximizes profit? Clearly justify your answer.

Solution: In order to maximize profit, we need to decide which of the points above could be local maxima. Since profit is revenue minus cost, we can find the sign of the derivative of profit by seeing where $MR-MC$ is positive and negative. Since $MR > MC$ for when the number of users is less than 450,000 and between 580,000 and 800,000 users, and $MR < MC$ everywhere else, 450 thousand users and 800 thousand users are local maxima of the profit function. In addition, since the area between the marginal cost and marginal revenue curves is greater between 450 and 580 (and counted negatively) than the area between 580 and 800, at 450 thousand users profit will be maximized.