

8. [12 points] Below is a table of values for the function $t(y)$ which gives the number of tweets per day, in millions, on the social media website Twitter, y years after January 1, 2007. For this problem assume $t(y)$ is an increasing function.

year y	0	1	2	3	4
millions of tweets per day $t(y)$	0.005	0.3	2.5	35	50

- a. [4 points] Using the table, estimate the expression

$$365 \int_1^4 t(y) dy$$

using a left-hand Riemann sum. Please write all of the terms in the sum for full credit.

Solution: To estimate this integral, we will use a left hand sum with 3 subdivisions. Since $n = 3$, $\Delta y = (4 - 1)/3 = 1$. Therefore, the left hand sum is

$$365 \int_1^4 t(y) dy \approx 365(t(1) \cdot 1 + t(2) \cdot 1 + t(3) \cdot 1) = 365(37.8) = 13,797 \text{ million tweets}$$

- b. [4 points] Give a practical interpretation of the expression $365 \int_1^4 t(y) dy$.

Solution: Since $t(y)$ gives tweets per day when you input a year, the units on the definite integral are (millions of tweets per day)(year). When we multiply by 365 days per year, we have the units of $365 \int_1^4 t(y) dy$ are millions of tweets. So the definite integral represents the total number tweets in millions that appeared on Twitter between January 1, 2008 and January 1, 2011.

- c. [4 points] Suppose $T(y)$ is the total number of tweets, in millions, y years after January 1, 2007. If $T(3) = 9797$, estimate the total number of tweets between January 1, 2007 and January 1, 2011. Indicate what method you use to obtain your estimate and be sure to show your work.

Solution: By the fundamental theorem of calculus, we know that

$$T(y) - T(3) = 365 \int_3^y t(w) dw.$$

In order to find $T(4)$, we need to compute the definite integral $\int_3^4 t(w) dw$. From the table above, we can use a left hand sum to compute $365 \int_3^4 t(w) dw = 365(35 \cdot 1) = 12775$. From our formula, we get that $T(4) = 12775 + T(3) = 22572$ million tweets. If we use a right hand sum to compute the value of the integral, we get $365 \int_3^4 t(w) dw = 365(50 \cdot 1) = 18250$ for a total of $T(4) = 28047$ million tweets.