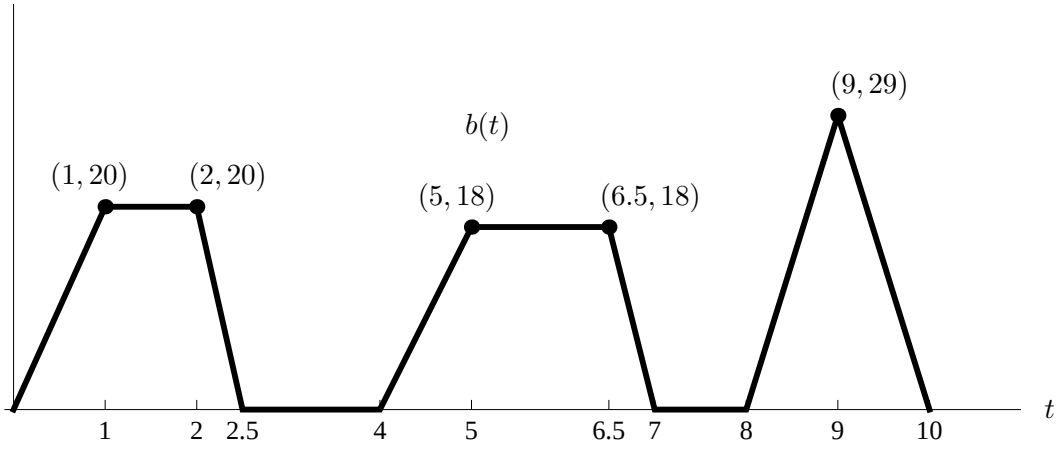
10. [12 points] Byron is blowing up a balloon. The rate at which he is blowing air into the balloon at time t is $b(t)$ cubic inches per second, graphed below. When $t = 0$, the balloon is empty.



a. [2 points] How much air has Byron blown into the balloon after 3 seconds?

Solution:

$$\int_0^3 b(t)dt = 10 + 20 + 5 = 35,$$

so after 3 seconds he has blown 35 cubic inches of air into the balloon.

After 3 seconds, the balloon springs a leak, and the air leaks out at a constant rate of r cubic inches per second.

b. [4 points] How much air is in the balloon 8 seconds after Byron started blowing it up? Your answer will involve r.

Solution:

$$\int_0^8 b(t)dt - 5r = 35 + 9 + 27 + 4.5 - 5r = 75.5 - 5r,$$

so after 8 seconds, there are $75.5 - 5r$ cubic inches of air in the balloon.

c. [3 points] Let $B(t)$ be the amount of air in the balloon after t seconds. Suppose $B(t)$ has a critical point at $t = 8.25$. Find r.

Solution: $B'(8.25) = 0$, which means $b(8.25) - r = 0$. We know $b(8.25) = \frac{29}{4}$, so $r = \frac{29}{4} = 7.25$ cubic inches per second.

d. [3 points] Is the critical point at $t = 8.25$ a local maximum, local minimum, or neither? Briefly explain.

Solution: At $t = 8.25$, $b(t)$ changes from being less than r to greater than r, so $B'(t)$ goes from negative to positive. Thus, by the first derivative test, the critical point is a local minimum.