1. [11 points] At a recent UM football game, a football scientist was measuring the excitement density, $E(x)$, in cheers per foot, in a one hundred foot row of the football stadium where x is the distance in feet from the beginning of the row. He took measurements every twenty feet and the data is recorded in this table.

x	0	20	40	60	80	100
$E(x)$	30	24	19	16	13	7

Assume for this problem that $E(x)$ is a decreasing function for $0 \le x \le 100$.

a. [6 points] Write a right sum and a left sum which approximate the total cheers in the row. Be sure to write all of the terms for each sum.

Solution:
LEFT $= 20(30) + 20(24) + 20(19) + 20(16) + 20(13) = 2040$
RIGHT $= 20(24) + 20(19) + 20(16) + 20(13) + 20(7) = 1580$

b. [2 points] Indicate whether the right and left sums are overestimates or underestimates for the total number of cheers in the row.

The right sum is an **overestimate** **underestimate**

The left sum is an **overestimate** **underestimate**

c. [3 points] How many measurements must the scientist take to guarantee that the left sum approximates the total number of cheers in the row within 5 cheers of the actual number?

Solution: The actual number of cheers is somewhere in between the left and right estimates because $E(x)$ is decreasing. So we want the difference between the left and right sums to be less than or equal to 5. If n is the number of subintervals used in the estimates, we have

$$|\text{Left} - \text{Right}| = |E(0) - E(100)| \cdot \frac{100 - 0}{n} = \frac{2300}{n}$$

We need $\dfrac{2300}{n} \le 5$, which is true if $n \ge 460$. We need 460 subintervals in our left sum estimate, so we need at least 460 measurements.