8. [11 points] A basketball player is running sprints in Crisler Center. She begins in the middle of the "M" at the center of the court and runs north and south. Her velocity, in meters per second, for the first 9 seconds is $v(t) = t\sin\left(\frac{\pi}{3}t\right)$, where t is the number of seconds since she started running. She is running north when $v(t)$ is positive and south when $v(t)$ is negative.

a. [3 points] Show that the function

$$f(t) = \frac{9}{\pi^2}\sin\left(\frac{\pi}{3}t\right) - \frac{3}{\pi}t\cos\left(\frac{\pi}{3}t\right)$$

is an antiderivative of $v(t)$.

Solution: To show that $f(t)$ is an antiderivative of $v(t)$, we need to show that $f'(t) = v(t)$.

$$\begin{aligned} f'(t) &= \frac{9}{\pi^2}\cos\left(\frac{\pi}{3}t\right)\frac{\pi}{3} - \frac{3}{\pi}\left(\cos\left(\frac{\pi}{3}t\right) - t\sin\left(\frac{\pi}{3}t\right)\frac{\pi}{3}\right) \\ &= \frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) - \frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + t\sin\left(\frac{\pi}{3}t\right) \\ &= t\sin\left(\frac{\pi}{3}t\right) \end{aligned}$$

b. [3 points] Where on the court is the player after the 9 seconds? Show all your work and give your answer in exact form (no decimal approximations).

Solution: The player starts at the "M" at the center of the court. The player's change in distance is equal to

$$\begin{aligned} \int_0^9 v(t)\,dt &= f(9) - f(0) \\ &= \left(\frac{9}{\pi^2}\sin\left(\frac{\pi}{3}(9)\right) - \frac{3}{\pi}(9)\cos\left(\frac{\pi}{3}(9)\right)\right) - \left(\frac{9}{\pi^2}\sin\left(\frac{\pi}{3}(0)\right) - \frac{3}{\pi}(0)\cos\left(\frac{\pi}{3}(0)\right)\right) \\ &= \frac{27}{\pi} \end{aligned}$$

So the player is $\frac{27}{\pi}$ meters north of the center of the court.

c. [5 points] What is the total distance traveled by the player in the 9 seconds? Show all your work and give your answer in exact form (no decimal approximations).

Solution: Note that $v(t)$ is positive for $0 < t < 3$, negative for $3 < t < 6$, and positive for $6 < t < 9$. Then the total distance traveled by the player is

$$\begin{aligned} \int_0^9 |v(t)|\,dt &= \int_0^3 v(t)\,dt - \int_3^6 v(t)\,dt + \int_6^9 v(t)\,dt \\ &= (f(3) - f(0)) - (f(6) - f(3)) + (f(9) - f(6)) \\ &= \frac{9}{\pi} - \left(-\frac{27}{\pi}\right) + \frac{45}{\pi} \\ &= \frac{81}{\pi} \end{aligned}$$

So the player traveled a distance of $\frac{81}{\pi}$ meters.