

9. [10 points] A team of engineers at the university has built a submarine to explore Lake Michigan. The engineers need to keep track of the temperature of the water outside the submarine in order to correctly regulate the engine temperature. The team is working with the following two functions:

- $f(t)$  is the depth of the submarine (in meters)  $t$  seconds after the submarine begins its descent. Assume  $f(t)$  is invertible on its domain.
- $g(d)$  be the temperature (in degrees Celsius) at a depth of  $d$  meters below the surface of Lake Michigan. Assume  $g(d)$  is invertible on its domain.

Circle the correct answer to each of the following questions. There is exactly one correct answer for each question.

- a. [2 points] Which expression is equal to the time in seconds at which the temperature of the water outside the submarine is 40 degrees?

$$f^{-1}(40) \quad g^{-1}(40) \quad \boxed{f^{-1}(g^{-1}(40))} \quad g^{-1}(f^{-1}(40))$$

- b. [2 points] Three minutes after it begins its descent, the submarine is at a depth of 45 meters. In the next second, the submarine descends approximately 2 meters. Which of the following equations is most likely to be true?

$$\boxed{f'(180) = 2} \quad f'(180) = 47 \quad f'(45) = 2 \quad f'(45) = 47$$

- c. [2 points] When the submarine is 50 meters below the surface, the temperature of the water outside the submarine will decrease by about 0.2 degrees in the next second. Which of the following equations is most likely to be true?

$$\boxed{g'(50)f'(f^{-1}(50)) = -0.2} \quad g'(50) = -0.2 \quad g'(f^{-1}(50)) = -0.2 \quad g'(g(50)) = -0.2$$

- d. [2 points] Which expression is equal to the average temperature outside the submarine during the first 10 seconds of the descent?

$$\frac{1}{10} \int_0^{10} g(x) dx \quad \boxed{\frac{1}{10} \int_0^{10} g(f(x)) dx} \quad \frac{1}{10} \int_0^{10} g'(x) dx \quad \frac{1}{10} \int_0^{10} g'(f(x)) dx$$

- e. [2 points] Which expression is equal to the change in temperature during the second minute of the submarine's descent?

$$\int_{60}^{120} g(f(x)) dx \quad \int_{60}^{120} g'(x) dx \quad \int_{60}^{120} g'(f(x)) dx \quad \boxed{\int_{60}^{120} g'(f(x))f'(x) dx}$$