7. [10 points] A history professor gives a 60 minute lecture, while one eager undergraduate student takes notes by typing what the professor says, word for word. Unfortunately, the student cannot always type as quickly as the professor is speaking.
Functions $p$ and $u$ are defined as follows. When $t$ minutes have passed since the start of the lecture, the professor is speaking at a rate of $p(t)$ words per minute ( wpm ) while the undergraduate student is typing at a rate of $u(t)$ words per minute (wpm). Shown below are graphs of $y=p(t)$ (dashed) and $y=u(t)$ (solid).

a. [2 points] How many minutes after the start of the lecture is the student typing most quickly?


#### Abstract

Answer: b. [3 points] Write a definite integral equal to the number of words the student types between the start of the lecture and the time the professor reaches the 600 th word of the lecture. You do not need to evaluate the integral.


#### Abstract

Answer: c. [3 points] How many minutes after the start of the lecture is the student furthest behind in typing up the lecture? (In other words, after how many minutes is the difference between the total number of words the professor has spoken and the total number of words the student has typed the greatest?)


Answer:
d. [2 points] What is the average rate, in words per minute, at which the professor is speaking between $t=40$ and $t=60$ ?

