9. [8 points] Consider the family of functions given by

$$I(t) = \frac{At^2}{B + t^2}$$

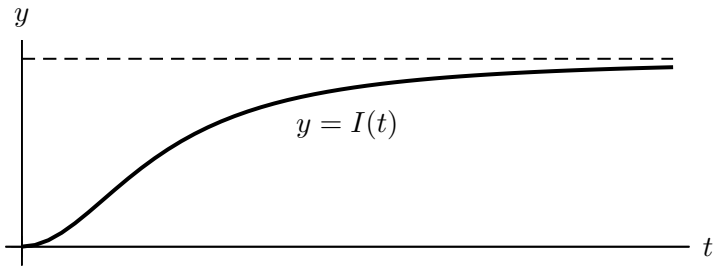
where A and B are positive constants. Note that the first and second derivatives of $I(t)$ are

$$I'(t) = \frac{2ABt}{(B + t^2)^2} \qquad \text{and} \qquad I''(t) = \frac{2AB(B - 3t^2)}{(B + t^2)^3}.$$

a. [2 points] Find $\lim_{t\to\infty} I(t)$. *Your answer may include the constants A and/or B.*

Answer: $\lim_{t\to\infty} I(t) =$ _____

A researcher studying the ice cover over Lake Michigan throughout the winter proposes that for appropriate values of A and B, the function $I(t)$ is a good approximation for the number of thousands of square miles of Lake Michigan covered by ice t days after the start of December. For such values of A and B, a graph of $y = I(t)$ for $t \geq 0$ is shown below.



Based on observations, the researcher chooses values of the parameters A and B so that the following are true.

- $y = 21$ is a horizontal asymptote of the graph of $y = I(t)$.
- $I(t)$ is increasing the fastest when $t = 25$.

b. [6 points] Find the values of A and B for the researcher's model.
Remember to show your work carefully.

Answer: $A =$ _____ and $B =$ _____