1. [13 points]
The graph of a function \( h(x) \) is shown on the right. The area of the shaded region \( A \) is 4, and \( h(x) \) is piecewise linear for \( 3 \leq x \leq 6 \).

Compute each of the following. If there is not enough information to compute a value exactly, write NOT ENOUGH INFO.

a. [2 points] Find \( \int_0^3 (h(x) + 2) \, dx \).

**Solution:**
\[
\int_0^3 (h(x) + 2) \, dx = \int_0^3 h(x) \, dx + \int_0^3 2 \, dx = 4 + (3)(2) = 10 \, .
\]

**Answer:** \( \int_0^3 (h(x) + 2) \, dx = 10 \)

b. [2 points] Find the average value of \( h(x) \) on the interval \( [0, 4] \).

**Solution:** The average value is given by
\[
\frac{1}{4-0} \int_0^4 h(x) \, dx = \frac{1}{4} (4-1) = \frac{3}{4} \, .
\]

**Answer:** \( \frac{3}{4} \)

c. [3 points] Let \( J(x) = \sin(\pi h(x)) \). Find \( J'(3.5) \).

**Solution:** Since \( J'(x) = \cos(\pi h(x)) \pi h'(x) \), we have that
\[
J'(3.5) = \cos(\pi h(3.5)) \pi h'(3.5) = \cos(-\pi) \pi (-2) = 2\pi \, .
\]

**Answer:** \( J'(3.5) = 2\pi \)

d. [3 points] Let \( H(x) \) be an antiderivative of \( h(x) \) with \( H(4) = 5 \). Find an equation for the tangent line to the graph of \( H(x) \) at \( x = 4 \).

**Solution:** Since \( H'(4) = h(4) = -2 \) and \( H(4) = 5 \), the tangent line to the graph of \( H(x) \) at \( x = 4 \) is given by \( y - 5 = -2(x-4) \).

**Answer:** \( y = 5 - 2(x-4) \) or \( y = 13 - 2x \)

e. [3 points] Let \( g(x) = e^x \). Find \( \int_6^7 (g(x)h'(x) + g'(x)h(x)) \, dx \).

**Solution:** Note that \( \frac{d}{dx} (g(x)h(x)) = g(x)h'(x) + g'(x)h(x) \). By the Fundamental Theorem of Calculus, the above integral is therefore equal to \( g(7)h(7) - g(6)h(6) = 2e^7 - 1e^6 \).

**Answer:** \( \int_6^7 (g(x)h'(x) + g'(x)h(x)) \, dx = 2e^7 - e^6 \)