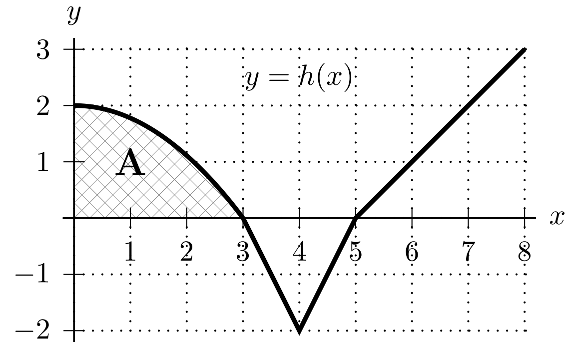
1. [13 points]

The graph of a function $h(x)$ is shown on the right. The area of the shaded region A is 4, and $h(x)$ is piecewise linear for $3 \le x \le 6$.



Compute each of the following. If there is not enough information to compute a value exactly, write NOT ENOUGH INFO.

a. [2 points] Find $\int_0^3 (h(x) + 2)\, dx$.

Solution:
$$\int_0^3 (h(x)+2)\, dx = \int_0^3 h(x)\, dx + \int_0^3 2\, dx = 4 + (3)(2) = 10\,.$$

Answer: $\int_0^3 (h(x)+2)\, dx =$ 10

b. [2 points] Find the average value of $h(x)$ on the interval $[0, 4]$.

Solution: The average value is given by $\frac{1}{4-0}\int_0^4 h(x)\, dx = \frac{1}{4}(4-1) = \frac{3}{4}$.

Answer: $\frac{3}{4}$

c. [3 points] Let $J(x) = \sin(\pi h(x))$. Find $J'(3.5)$.

Solution: Since $J'(x) = \cos(\pi h(x))\pi h'(x)$, we have that
$$J'(3.5) = \cos(\pi h(3.5))\pi h'(3.5) = \cos(-\pi)\pi(-2) = 2\pi\,.$$

Answer: $J'(3.5) =$ 2π

d. [3 points] Let $H(x)$ be an antiderivative of $h(x)$ with $H(4) = 5$. Find an equation for the tangent line to the graph of $H(x)$ at $x = 4$.

Solution: Since $H'(4) = h(4) = -2$ and $H(4) = 5$, the tangent line to the graph of $H(x)$ at $x = 4$ is given by $y - 5 = -2(x-4)$.

Answer: $y = 5 - 2(x-4)$ **or** $y = 13 - 2x$

e. [3 points] Let $g(x) = e^x$. Find $\int_6^7 \big(g(x)h'(x) + g'(x)h(x)\big)\, dx$.

Solution: Note that $\frac{d}{dx}(g(x)h(x)) = g(x)h'(x) + g'(x)h(x)$. By the Fundamental Theorem of Calculus, the above integral is therefore equal to $g(7)h(7) - g(6)h(6) = 2e^7 - 1e^6$.

Answer: $\int_6^7 \big(g(x)h'(x) + g'(x)h(x)\big)\, dx =$ $2e^7 - e^6$