2. [8 points] A car is traveling on a long straight road. The driver suddenly realizes that there is a stop sign exactly 40 feet in front of the car and immediately hits the brakes. The car's velocity decreases for the next two seconds as the car slows to a stop.
Let $v(t)$ be the velocity of the car, in feet per second, t seconds after the driver hits the brakes. Some values of the function v are shown in the table below.

t	0	0.5	1	1.5	2
$v(t)$	40	32	23	12	0

a. [2 points] Estimate the car's acceleration 0.25 seconds after the driver hits the brakes. *Remember to show your work and include units.*

Solution: The acceleration at $t = 0.25$ can be approximated by the difference quotient

$$\frac{v(0.5) - v(0)}{0.5 - 0} = \frac{32 - 40}{0.5 - 0} = -16 \text{ ft/s}^2 .$$

Answer: -16 **ft/s^2**

b. [3 points] Based on the information in the table above, does the car first stop before, after, or at the stop sign? Or, is there not enough information to make this determination? Briefly explain your reasoning.

Answer: (Circle one choice.)

Before the sign After the sign At the sign $\boxed{\text{Not enough info}}$

Reasoning:

Solution: The distance (in feet) travelled by the car before it stops is $\int_0^2 v(t)\,dt$. Since $v(t)$ is decreasing for $0 \le t \le 2$, the left-hand sum gives an overestimate for $\int_0^2 v(t)\,dt$, while the right-hand sum gives an overestimate.

Since

$$\text{Left-hand sum} = (0.5)(40) + (0.5)(32) + (0.5)(23) + (0.5)(12) = 53.5$$

and

$$\text{Right-hand sum} = (0.5)(32) + (0.5)(23) + (0.5)(12) + (0.5)(0) = 33.5,$$

we cannot determine whether or not $\int_0^2 v(t)\,dt$ is greater than or less than 40. (The estimates 53.5 and 33.5 are the best overestimate and underestimate, respectively, that we can make of the actual distance travelled based on the data we have.)

c. [3 points] How often would speedometer readings need to be taken so that the resulting left-hand Riemann sum approximates the actual distance traveled between $t = 0$ and $t = 2$ seconds to within 1 foot?

Solution: Since $v(t)$ is decreasing, the difference between the left-hand sum and the actual value is at most the difference between the left- and right-hand sums.
When readings are taken every Δt seconds, the difference between the two sums is $(\Delta t)|v(2) - v(0)|$. We choose Δt so that $1 \ge \Delta t|v(2) - v(0))| = \Delta t(40)$. Thus, $\Delta t \le 1/40$.

Answer: Readings would need to be taken once every 1/40 **or** 0.025 seconds.