3. [12 points] Oren plans to grow kale on his community garden plot, and he has determined that he can grow up to 160 bunches of kale on his plot. Oren can sell the first 100 bunches at the market and any remaining bunches to wholesalers. The revenue in dollars that Oren will take in from selling b bunches of kale is given by

$$R(b) = \begin{cases} 6b & \text{for } 0 \le b \le 100 \\ 4b + 200 & \text{for } 100 < b \le 160. \end{cases}$$

a. [2 points] Use the formula above to answer each of the following questions.

i. What is the price (in dollars) that Oren will charge for each bunch of kale he sells at the market?

Answer: \$6

ii. What is the price (in dollars) that Oren will charge for each bunch of kale he sells to wholesalers?

Answer: \$4

For $0 \le b \le 160$, it will cost Oren $C(b) = 20 + 3b + 24\sqrt{b}$ dollars to grow b bunches of kale.

b. [1 point] What is the fixed cost (in dollars) of Oren's kale growing operation?

Answer: \$20

c. [4 points] At what production level(s) does Oren's marginal revenue equal his marginal cost?

Solution: Oren's marginal revenue is $R'(b) = 6$ for $0 < b < 100$ and $R'(b) = 4$ for $100 < b < 160$. His marginal cost is $C'(b) = 3 + 12/\sqrt{b}$.
Thus, $R'(b) = C'(b)$ for $b = 16$ and $b = 144$.

Answer: **at 16 bunches and 144 bunches**

d. [5 points] Assuming Oren can grow up to 160 bunches of kale, how many bunches of kale should he grow in order to maximize his profit, and what is the maximum possible profit? *You must use calculus to find and justify your answer. Be sure to provide enough evidence to justify your answer fully.*

Solution: Since Oren's profit function, $\pi(b) = R(b) - C(b)$, is continuous on $0 \le b \le 160$, it has a global maximum (by the Extreme Value Theorem) and the global maximum occurs at a critical point or an endpoint.
The critical points of $\pi(b)$ occur when $\pi'(b) = 0$ (at $b = 16$ and 144 (when MR=MC)), and when $\pi'(b)$ is undefined (at $b = 100$).
We check the value of $\pi(b)$ at the critical points and end points:
$\pi(0) = -20$, $\pi(16) - -68$, $\pi(100) = 40$, $\pi(144) = 36$, and $\pi(160) \approx 36.42$, and conclude that the maximum occurs at $b = 100$, with a resulting maximum profit of \$40.

Answer: bunches of kale: 100 and max profit: \$40