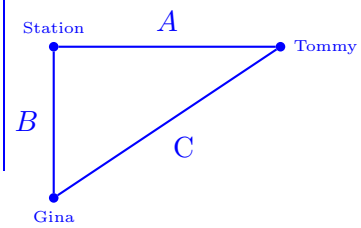
5. [10 points] Tommy and Gina were friends in high school but then went to college in different parts of the country. They thought they were going to see each other in Springfield over the December break, but their schedules didn't match up. In fact, it turns out that Tommy is leaving on the same day that Gina is arriving.

Shortly before Gina's train arrives in Springfield, she sends a text to Tommy to see where he is, and Tommy sends a text response to say that, sadly, his train has already left. At the moment Tommy sends his text, he is 20 miles due east of the center of the train station and moving east at 30 mph while Gina is 10 miles due south of the train station and moving north at 50 mph.

a. [2 points] What is the distance between Gina and Tommy at the time Tommy sends his text? *Remember to include units.*

Solution:



Let A, B, and C be the distances (in miles) as labeled in the drawing on the left. Then by the Pythagorean Theorem, $C^2 = A^2 + B^2$. When Tommy sends his text, $A = 20$ and $B = 10$ so we conclude that Gina and Tommy are $\sqrt{20^2 + 10^2} = \sqrt{500} = 10\sqrt{5} \approx 22.4$ miles apart.

Answer: $\sqrt{500} = 10\sqrt{5} \approx 22.4$ **miles**

b. [6 points] When Tommy sends his text, are he and Gina moving closer together or farther apart? How quickly? *You must show your work clearly to earn any credit. Remember to include units.*

Solution: With the notation from part (a), we have that

$$2C\frac{dC}{dt} = 2A\frac{dA}{dt} + 2B\frac{dB}{dt}.$$

When Tommy sends his text, we know that $\dfrac{dA}{dt} = 30$ and $\dfrac{dB}{dt} = -50$. Thus,

$$\frac{dC}{dt} = \frac{A\frac{dA}{dt} + B\frac{dB}{dt}}{C} = \frac{(20)(+30) + (10)(-50)}{\sqrt{500}} = 2\sqrt{5} \approx 4.47.$$

Since at this time the sign of $\frac{dC}{dt}$ is positive, Gina and Tommy are getting farther apart.

Answer: Tommy and Gina are getting (circle one) CLOSER TOGETHER FARTHER APART

at a rate of $2\sqrt{5} \approx 4.47$ **mph**

c. [2 points] Let $J(t)$ be the distance between Gina and Tommy t hours after Tommy sends his text. Use the local linearization of $J(t)$ at $t = 0$ to estimate the distance between Gina and Tommy 0.1 hours after Tommy sends his text. *Remember to show your work carefully.*

Solution: This local linearization is $L(t) = J(0) + J'(0)(t) = 10\sqrt{5} + 2t\sqrt{5}$. So the distance between Tommy and Gina 0.1 seconds after Tommy sends his text will be approximately $L(0.1) = 10\sqrt{5} + (0.1)(2\sqrt{5}) = 10.2\sqrt{5} \approx 22.8$ miles.

Answer: $10.2(\sqrt{5}) \approx 22.8$ **miles**