**6**. [7 points] Consider the family of functions given by

$$f(x) = \frac{ax}{e^{0.5(bx)^2}}$$

where $a$ and $b$ are constants with $a > 1$ and $b > 1$.
Note that the derivative and second derivative of $f(x)$ are

$$f'(x) = \frac{a(1 - b^2x^2)}{e^{0.5(bx)^2}} \qquad \text{and} \qquad f''(x) = \frac{ab^2x(b^2x^2 - 3)}{e^{0.5(bx)^2}}.$$

Find all global extrema of $f(x)$ on the interval $\left[\frac{1}{4b}, \infty\right)$. If there are none of a particular type, write NONE.
*You must use calculus to find and justify your answers. Be sure to provide enough evidence to justify your answers fully.*

*Solution:* In the domain $\left[\frac{1}{4b}, \infty\right)$ $f(x)$ has one critical point, which is at $x = \frac{1}{b}$. (Note that $f'(x) = 0$ for $x = \pm 1/b$ and $x = -1/b$ is not in our domain.)
Since

$$f''\left(\frac{1}{b}\right) = \frac{ab(-2)}{e^{0.5}} < 0$$

$x = 1/b$ is a local maximum of $f(x)$.
Since $f(x)$ has exactly one critical point in our domain and it is a local maximum, $x = 1/b$ is a global maximum. (Alternatively, one can note that sign of $f'(x)$ changes from positive to negative at $x = 1/b$ and does not change sign again in the interval (since no other critical points), so $f(x)$ is increasing from $\frac{1}{4b}$ to $\frac{1}{b}$ and always decreasing thereafter.)
The function $f(x)$ has no global minimum on the interval $[\frac{1}{4b}, \infty)$ since $\lim_{x\to\infty} f(x) = 0$, but $f(x) > 0$ on $[\frac{1}{4b}, \infty)$. (Alternatively, one can note as above that $f(x)$ is increasing from $\frac{1}{4b}$ to $\frac{1}{b}$ and always decreasing thereafter so we compare $f(\frac{1}{4b})$ (which is strictly positive) to $\lim_{x\to\infty} f(x) = 0$.)

**Answer:** global max(es) at $x =$ $1/b$

**Answer:** global min(s) at $x =$ NONE