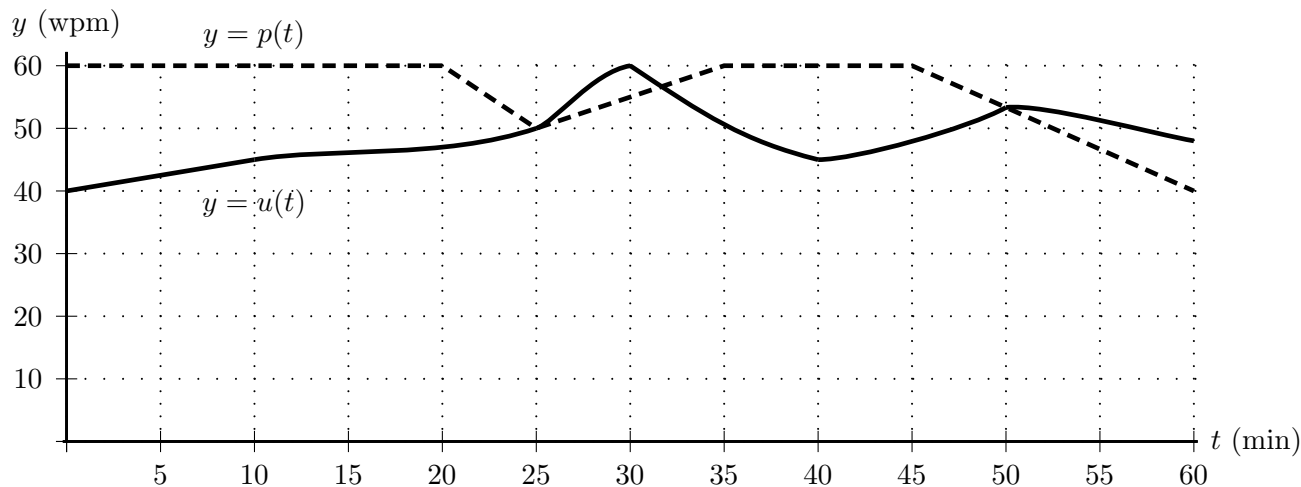


7. [10 points] A history professor gives a 60 minute lecture, while one eager undergraduate student takes notes by typing what the professor says, word for word. Unfortunately, the student cannot always type as quickly as the professor is speaking.

Functions p and u are defined as follows. When t minutes have passed since the start of the lecture, the professor is speaking at a rate of $p(t)$ words per minute (wpm) while the undergraduate student is typing at a rate of $u(t)$ words per minute (wpm). Shown below are graphs of $y = p(t)$ (dashed) and $y = u(t)$ (solid).



- a. [2 points] How many minutes after the start of the lecture is the student typing most quickly?

Solution: The student is typing most quickly when $u(t)$ is maximized, which is at $t = 30$.

Answer: 30 minutes

- b. [3 points] Write a definite integral equal to the number of words the student types between the start of the lecture and the time the professor reaches the 600th **w**ord of the lecture. You do not need to evaluate the integral.

Solution: The professor reaches the 600th word of the lecture 10 minutes after the start of the lecture (since the professor is speaking at a constant rate of 60 wpm for the first 20 minutes). The total number of words the student has typed by then is $\int_0^{10} u(t) dt$.

Answer: $\int_0^{10} u(t) dt$

- c. [3 points] How many minutes after the start of the lecture is the student furthest behind in typing up the lecture? (In other words, after how many minutes is the difference between the total number of words the professor has spoken and the total number of words the student has typed the greatest?)

Solution: This is equal to the time t when the difference between the total area under the graph of $p(t)$ and the total area under the graph of $u(t)$ is the greatest. Local maxima of this difference occur at $t = 25$ and $t = 50$. Since the area between the two graphs from $t = 25$ to $t = 32$ is smaller than the area between the two graphs from $t = 32$ to $t = 50$, the difference between the number of words spoken and typed is greatest when $t = 50$.

Answer: 50 minutes

- d. [2 points] What is the average rate, in words per minute, at which the professor is speaking between $t = 40$ and $t = 60$?

Solution: This average rate is $\frac{1}{20} \int_{40}^{60} p(t) dt = \frac{1050}{20} = 52.5$ words per minute.

Answer: 52.5