8. [11 points] Suppose k and p are positive constants. Consider the function

$$R(x) = p - \ln(x^2 + k)\,.$$

a. [5 points] Use the limit definition of the derivative to write down an explicit expression for $R'(3)$.
Your answer should not include the letter R.
Do not attempt to evaluate or simplify the limit.

Answer: $R'(3) = \lim_{h \to 0} \dfrac{(p - \ln((3+h)^2 + k)) - (p - \ln(3^2 + k))}{h}$

b. [4 points] Write out all the terms for the right-hand Riemann sum with three subdivisions of equal length which approximates the integral

$$\int_1^{13} R(x)\,dx\,.$$

Your answer should not include the letter R but may involve k and/or p.

Solution: This right hand sum is given by

$$4R(5) + 4R(9) + 4R(13) = 4(p - \ln(25 + k)) + 4(p - \ln(81 + k)) + 4(p - \ln(169 + k))\,.$$

c. [2 points] Is the right-hand Riemann sum with three subdivisions of equal length from part (b) an overestimate or an underestimate of $\int_1^{13} R(x)\,dx$, or is there not enough information to make this determination? Briefly explain your reasoning.

Answer: (Circle one choice.)

Overestimate   **[Underestimate]**   Not enough info

Reasoning:

Solution: Since $R'(x) = -\frac{2x}{x^2+k}$ is negative for $x > 0$, $R(x)$ is decreasing on the interval $1 \le x \le 13$. Thus, the above right-hand Riemann sum is an underestimate for the integral.