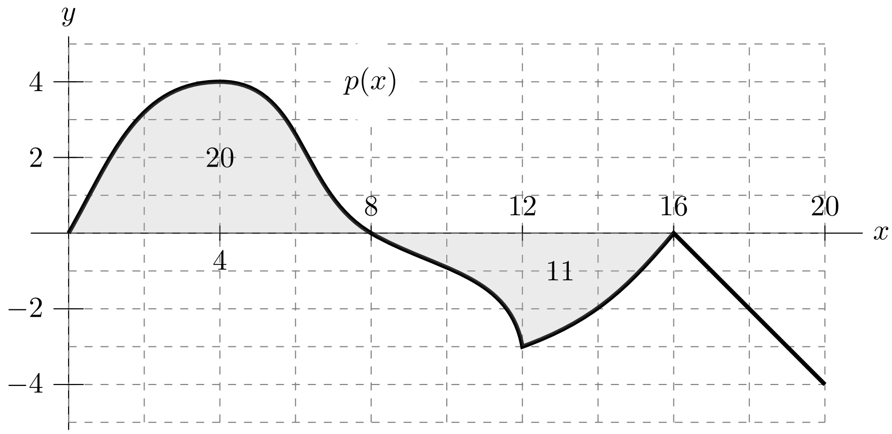
1. [12 points] Recall that a function h is odd if $h(-x) = -h(x)$ for all x. A portion of the graph of $p(x)$, an odd function, is shown below. Assume that the areas of the two shaded regions are 20 and 11, as indicated on the graph, and note that $p(x)$ is linear for $16 < x < 20$.



Remember to show your work throughout this problem.

a. [4 points] Compute the exact value of $\int_0^{20} (5 - 3p(x))\,dx$.

Solution: We have $$\int_0^{20} (5 - 3p(x))\,dx = \int_0^{20} 5\,dx - 3\int_0^{20} p(x)\,dx$$ $$= 100 - 3(20 - 11 - \frac{4\cdot 4}{2}) = 97.$$

Answer: 97

b. [2 points] Compute the exact value of $\int_4^8 p'(x)\,dx$.

Solution: By the Fundamental Theorem, we have $\int_4^8 p'(x)\,dx = p(8) - p(4) = 0 - 4 = -4$.

Answer: -4

c. [3 points] Find the average value of $p(x)$ on the interval $-16 \le x \le 8$.

Solution: The average value is given by $\frac{1}{8-(-16)}\int_{-16}^{8} p(x)\,dx$. Since p is odd, we have $\int_{-8}^{8} p(x)\,dx = 0$, and $\int_{-16}^{-8} p(x)\,dx = -\int_8^{16} p(x)\,dx$. Thus, the average value is

$$\frac{1}{24}\left(\int_{-16}^{8} p(x)\,dx\right) = \frac{1}{24}\left(\int_{-16}^{-8} p(x)\,dx + \int_{-8}^{8} p(x)\,dx\right) = \frac{1}{24}\left(-\int_8^{16} p(x)\,dx + 0\right) = \frac{11}{24}.$$

Answer: $\frac{11}{24}$

d. [3 points] Use a right Riemann sum with 3 equal subintervals to estimate $\int_{12}^{18} p(x)\,dx$. Write out all terms of the sum.

Solution:

$$2\,(p(14) + p(16) + p(18)) = 2(-2 + 0 + (-2)) = -8.$$

Answer: $\int_{12}^{18} p(x)\,dx \approx$ -8