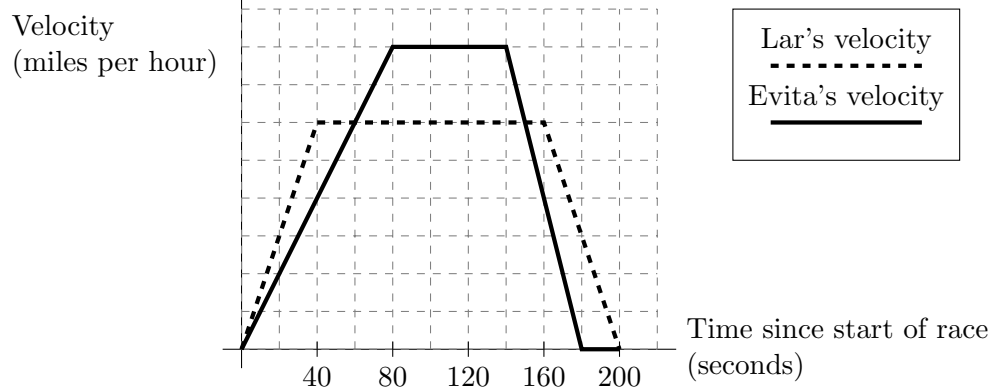


3. [12 points] Lar Getni and Evita Vired run a half-mile race. After the race, C.T. Latnem Adnuf receives the following graph of the two runners' velocities over the course of the race.



Unfortunately, whoever made the graph forgot to label the scale of the vertical axis, and C.T. needs your help to answer the following questions. You may assume that the horizontal grid lines are evenly spaced, but do not assume that the scales of the two axes are the same. You may also assume that both runners completed the race and then stopped running.

- a. [1 point] Who won the race?

**Answer:** Evita

- b. [2 points] During what time interval(s) was Lar ahead of Evita?

**Answer:**  $0 < t < 100$

- c. [2 points] During what time interval(s) was Lar running faster than Evita?

**Answer:**  $0 < t < 60$  and  $150 < t < 200$

- d. [4 points] What was the maximum speed (in miles per hour) attained by Lar? By Evita? *Remember to show your work.*

*Solution:* There are 48 boxes under the graph of Lar's (and also of Evita's) velocity. Let  $c$  denote the vertical dimension of a box, in miles per hour. The horizontal dimension of a box is 20 seconds, or  $\frac{20}{3600}$  hours. Since the race is  $\frac{1}{2}$  a mile long, and the area under the curve is equal to the distance traveled, we must have

$$48 \cdot c \cdot \frac{20}{3600} = \frac{1}{2}$$

so  $c = 1.875$ . Thus, the maximum speed attained by Lar is  $6 \cdot 1.875 = 11.25$  mph, and the maximum speed attained by Evita is  $8 \cdot 1.875 = 15$  mph.

**Answer:** Lar's max speed: 11.25 mph and Evita's max speed: 15 mph

- e. [3 points] Let  $v(t)$  (respectively,  $w(t)$ ) be Evita's (respectively, Lar's) velocity in miles per hour  $t$  seconds after the start of the race. Write an equation involving one or more integrals that expresses the following statement:

$N$  seconds after the start of the race, Evita is  $M$  miles ahead of Lar.

Your answer may involve  $v(t)$  and  $w(t)$ .

**Answer:**  $\frac{1}{3600} \int_0^N (v(t) - w(t)) dt = M$