4. [7 points] Below is a table showing some values of an invertible and differentiable function m.

t	-0.11	-0.03	0	0.02	0.5	0.98	1	1.06	1.12
$m(t)$	1.548	1.423	1.000	0.721	0	-2.367	-2.441	-2.675	-2.913

Find the value of each of the quantities below. If it is not possible to find the value exactly, find the best estimate you can given all of the information provided above.

a. [1 point] $\lim_{t\to 0} m(t)$

Solution: Since m is differentiable, it is continuous, so $\lim_{t\to 0} m(t) = m(0) = 1.000$.

Answer: 1.000

b. [2 points] $(m^{-1})'(1)$

Solution: $(m^{-1})'(1) \approx \dfrac{m^{-1}(0.721) - m^{-1}(1)}{0.721 - 1} = \dfrac{0 - 0.02}{1 - 0.721} \approx -0.0717$

(We can also consider $\frac{m^{-1}(1.423)-m^{-1}(1)}{1.423-1} = \frac{-0.03-0}{1.423-1} \approx -0.0709$

or $\frac{m^{-1}(1.423)-m^{-1}(0.721)}{1.423-0.721} = \frac{-0.03-0.02}{1.423-0.721} = \frac{-0.03-0}{1.423-1} \approx -0.0712$.)

Answer: **Approximately** -0.07

c. [1 point] $\lim_{u\to 0} \dfrac{m(1+u) - m(1)}{u}$

Solution: $\lim_{u\to 0} \dfrac{m(1+u) - m(1)}{u} \approx \dfrac{m(1+(-0.02)) - m(1)}{-0.02} = \dfrac{-2.367 - (-2.441)}{-0.02} = -3.7$

Answer: **Approximately** -3.7

Below is a table showing some values of another differentiable function n.
Assume that $n'(t)$ is continuous on the interval $[-0.1, 1.1]$.

t	-0.1	0	0.1	0.9	1	1.1
$n(t)$	2	-8	5	2	-2	3

Find the <u>exact value</u> of each of the quantities below. If it is not possible to find the value exactly, write NOT POSSIBLE.

d. [1 point] The average rate of change of $n(t)$ on the interval $[-0.1, 1.1]$

Solution: $\dfrac{n(1.1) - n(-0.1)}{1.1 - (-0.1)} = \dfrac{3-2}{1.2} = \dfrac{1}{1.2} = \dfrac{5}{6}$.

Answer: $\frac{5}{6}$

e. [1 point] $\int_0^1 n'(t)\,dt$

Solution: By the Fundamental Theorem of Calculus, we have

$$\int_0^1 n'(t)dt = n(1) - n(0) = -2 - (-8) = 6.$$

Answer: 6

f. [1 point] $\int_0^1 n(t)\,dt$

Answer: NOT POSSIBLE