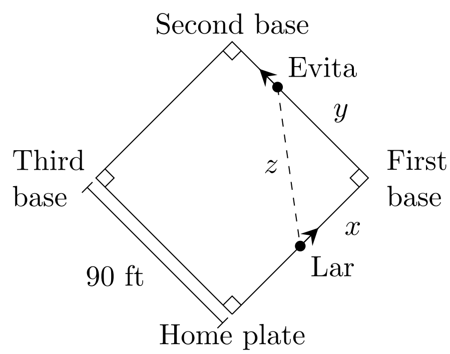
5. [9 points]
During the annual Srebmun Foyoj kickball game, Lar Getni kicks the ball and runs from home plate to first base, while Evita Vired runs from first base to second base.



Let x be the distance between Lar and first base, y be the distance between Evita and first base, and z be the distance between Lar and Evita, as shown in the diagram on the right. Note that the bases are arranged in a square and that the distance between consecutive bases is 90 feet.

At the moment when Lar is halfway from home plate to first base, Evita is two thirds of the way from first base to second base. At this moment, Lar is running at a speed of 32 ft/s, and Evita is running at a speed of 36 ft/s. The questions below all refer to this moment.

Throughout this problem, remember to show your work clearly, and include units in your answers.

a. [5 points] At the moment when Lar is halfway to first base, at what rate is the distance between Lar and Evita changing? Is the distance increasing or decreasing?

Solution: We need to find $\frac{dz}{dt}$ at the moment when Lar is halfway to first base. By the Pythagorean Theorem, we have $z^2 = x^2 + y^2$. Differentiating both sides of this equation with respect to time, we find that $2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$.

At the moment in question, we have $x = 45$ ft, $y = 60$ ft, $z = \sqrt{45^2 + 60^2} = 75$ ft, $\frac{dx}{dt} = -32$ ft/s, and $\frac{dy}{dt} = 36$ ft/s. So at this moment,

$$\frac{dz}{dt} = \frac{2(45)(-32) + 2(60)(36)}{2(75)} = 9.6 \text{ ft/s}.$$

Answer: The distance is (circle one) $\boxed{\text{INCREASING}}$ DECREASING

at a rate of 9.6 **ft/s** .

b. [4 points] At the moment when Lar is halfway to first base, at what rate is the area of the right triangle formed by Lar, Evita, and first base changing? Is the area increasing or decreasing?

Solution: The area of the triangle is given by $A = \frac{xy}{2}$. Differentiating both sides of this equation with respect to time, we find that $\frac{dA}{dt} = \frac{x\frac{dy}{dt} + y\frac{dx}{dt}}{2}$.

At the moment when Lar is halfway to first base, we therefore have

$$\frac{dA}{dt} = \frac{(45)(36) + (60)(-32)}{2} = -150 \text{ ft}^2\text{/s}.$$

Answer: The area is (circle one) INCREASING $\boxed{\text{DECREASING}}$

at a rate of 150 $\mathbf{ft^2/s}$.