8. [8 points] Elur Niahc keeps a bucket in his backyard. It contains water, and the water is two inches deep when a rainstorm starts. The storm lasts 20 minutes.

- Let h be the depth, in inches, of the water in the bucket.
- Let $V(h)$ be the volume, in gallons, of water in the bucket when the water is h inches deep. Assume that $V(h)$ is invertible and differentiable.
- Let $r(t)$ be the rate at which the volume of water in the bucket is increasing, measured in gallons per minute, t minutes after the storm starts. Assume that $r(t) > 0$ for the entire duration of the rainstorm.

For each of the questions below, circle the one best answer. No points will be given for ambiguous or multiple answers.

a. [2 points] Which of the following expressions represents the depth, in inches, of water in the bucket when the bucket contains 3 gallons of water?

i. $V(3)$ ii. $\boxed{V^{-1}(3)}$ iii. $2 + V(3)$ iv. $2 + V'(3)$

b. [2 points] Which of the following is the best interpretation of the equation $(V^{-1})'(3) = 0.4$?

i. The rate at which the depth of the water in the bucket is changing is increasing by 0.4 inches per minute when the bucket contains 3 gallons of water.

ii. During the third minute of the rainstorm, the volume of the water in the bucket increases by about 0.4 gallons.

iii. When the depth of the water in the bucket increases from 2.8 to 3 inches, the volume of the water increases by about 0.08 gallons.

iv. $\boxed{\text{When the volume of the water in the bucket is 3 gallons, the depth of the water is about 0.2 inches less than the depth will be when the volume is 3.5 gallons.}}$

c. [2 points] Which expression represents the volume, in gallons, of water in the bucket after the rainstorm ends?

i. $V\left(2 + \int_0^{20} r(t)\,dt\right)$ iii. $\int_0^{20} V(2 + r'(t))\,dt$ v. $2 + V(20)$

ii. $2 + \int_0^{20} r(t)\,dt$ iv. $\boxed{V(2) + \int_0^{20} r(t)\,dt}$ vi. $\int_0^{20} V'(t)\,dt$

d. [2 points] Which of the following represents the average rate of change of the volume, in gallons per minute, of the water in the bucket during the rainstorm?

i. $\dfrac{V(20) - V(0)}{20}$ iii. $\boxed{\dfrac{1}{20}\int_0^{20} r(t)\,dt}$

ii. $\dfrac{r(20) - r(0)}{20}$ iv. $\dfrac{1}{20}\int_0^{20} V(t)\,dt$