

9. [10 points] Suppose $g(x)$ is a function and $G(x)$ is an antiderivative of $g(x)$ such that $G(x)$ is defined and continuous on the entire interval $-4 \leq x \leq 6$. Portions of the graphs of g and G are shown below. Note the following:

- $g(x)$ is zero for $-1 \leq x \leq 0$.
- For $4 \leq x \leq 6$, the graph of $g(x)$ is the lower half of the circle of radius 1 centered at $(5, 0)$.
- For $0 \leq x \leq 1$, the graph of $G(x)$ is the top right quarter of the circle of radius 1 centered at the origin.

a. [4 points] Use the portions of both graphs shown on the right to complete the table below with the exact values of $G(x)$.

x	-3	-1	4	6
$G(x)$	3	1	4	$4 - \frac{\pi}{2}$

Solution: Note from the graph of G that $G(0) = 1$, $G(1) = 0$, and $G(2) = 2$. We find the values in the table by using the Fundamental Theorem of Calculus (and finding appropriate areas using the graph of $g(x)$).

$$G(4) = \int_2^4 g(x) dx + G(2) = 2 + 2 = 4.$$

$$G(6) = \int_4^6 g(x) dx + G(4) = -\frac{\pi}{2} + 4.$$

$$G(-1) = G(0) - \int_{-1}^0 f(x) dx = 1 - 0 = 1.$$

$$G(-3) = G(-1) - \int_{-3}^{-1} f(x) dx = 1 - (-2) = 3.$$

b. [6 points] Use the axes on the right to sketch the missing portions of the graphs of g and G over the interval $-4 \leq x \leq 6$.

Be sure that you pay close attention to each of the following:

- the values of $G(x)$ you found in part (a) above
- where G is/is not differentiable
- where G and g are increasing, decreasing, or constant
- the concavity of the graph of $y = G(x)$

