10. [10 points] Yukiko has a small orchard where she grows Michigan apples. After careful study last season, Yukiko found that the total cost, in dollars, of producing a bushels of apples can be modeled by

$$C(a) = -25500 + 26000e^{0.002a}$$

for $0 \leq a \leq 320$.

Qabil has promised to buy up to 100 bushels of apples for his famous apple ice cream. If Yukiko has any remaining apples, she has an agreement to sell them to Xanthippe's cider mill at a reduced price. Let $R(a)$ be the revenue generated from selling a bushels of apples. Then

$$R(a) = \begin{cases} 70a & \text{if } 0 \leq a \leq 100 \\ 2000 + 50a & \text{if } 100 < a \leq 320. \end{cases}$$

a. [1 point] How much will Xanthippe's cider mill pay per bushel?

Answer: _____ **$50** _____

b. [1 point] What is Yukiko's fixed cost?

Answer: _____ **$500** _____

c. [4 points] For what quantities of bushels of apples sold would Yukiko's marginal revenue equal her marginal cost? Write NONE if appropriate.

> *Solution:* Yukiko's marginal revenue is given by
>
> $$MR = \begin{cases} 70 & \text{if } 0 < a < 100 \\ 50 & \text{if } 100 < a < 320 \end{cases}$$
>
> and her marginal cost is $52e^{0.002a}$. We have $52e^{0.002a} = 70$ when $a \approx 148.63$, but this is greater than 100, so it is not in the correct domain. Also $52e^{0.002a} = 50$ when $a \approx -19.61$, which is also not in the domain. Thus there are no values of a where $MC = MR$.

Answer: _____ **None** _____

d. [4 points] Assuming Yukiko can produce up to 320 bushels of apples, how many bushels should she produce in order to maximize her profit, and what would that maximum profit be? You must use calculus to find and justify your answer. Make sure to provide enough evidence to justify your answer fully.

> *Solution:* Let $\pi(a) = R(a) - C(a)$ be the profit function. Note that $C(a)$ and $R(a)$ are continuous on this closed interval, so we can apply the Extreme Value Theorem. Since we found in the previous part that MC and MR are never equal, we only need to consider endpoints and points where $\pi'(a)$ does not exist. This happens when $q = 100$. Using the formulas we've been given, we find
>
> $$\pi(0) = -500$$
> $$\pi(100) \approx 743.53$$
> $$\pi(320) \approx -5808.50$$

Answer: bushels of apples: _____ **100** _____ and max profit: _____ **$743.53** _____