12. [8 points] Let W be the differentiable function given by

$$W(p) = \begin{cases} 4\ln(2) + 4\ln(-p) & \text{if } p \le -0.5 \\ 2\sin(4p^2 - 1) & \text{if } -0.5 < p < 0.5 \\ \dfrac{\arctan(2p-1)}{p^2} & \text{if } p \ge 0.5. \end{cases}$$

a. [4 points] Use the limit definition of the derivative to write an explicit expression for $W'(3)$. *Your answer should not involve the letter W. Do not evaluate or simplify the limit.* Please write your final answer in the answer box provided below.

Answer: $W'(3) = \lim_{h\to 0} \dfrac{\frac{\arctan(2\cdot 3-1+h)}{(3+h)^2} - \frac{\arctan(2\cdot 3-1)}{9}}{h}$

b. [4 points] With W as defined above, consider the function g defined by

$$g(t) = \begin{cases} ct + k & \text{if } t \le 0 \\ W(-e^t) & \text{if } t > 0 \end{cases}$$

for some constants c and k. Find all values of c and k so that $g(t)$ is differentiable.
Show your work carefully, and leave your answers in exact form.
If no such values of c and/or k exist, write NONE in the appropriate answer blank and be sure to justify your reasoning.

Solution: Note that for $t > 0$, $g(t) = 4\ln(2) + 4t$, so

$$g'(t) = \begin{cases} c & \text{for } t < 0 \\ 4 & \text{for } t > 0 \end{cases}$$

(Alternatively, $g'(t) = W'(-e^t)\cdot -e^t$.) Since we are told that W is differentiable, we need only to find values so that g is differentiable at $t = 0$.
In order for g to be differentiable, we need to find values of c and k so that $\lim_{t\to 0^-} g(t) = \lim_{t\to 0^+} g(t)$ and $\lim_{t\to 0^-} g'(t) = \lim_{t\to 0^+} g'(t)$.

The first equation is true when $c\cdot 0 + k = W(-e^0) = W(-1)$. Note that since $-1 < -0.5$, we have $W(-1) = 4\ln(2) + 4\ln(-(-1)) = 4\ln(2)$, so $k = 4\ln(2)$.

The second equation is true when $c = -W'(-1)$. Near $p = -1$ we have $W'(p) = 4$. Therefore, we need $c = 4$.

Answer: $c =$ **4** and $k =$ $4\ln(2)$