12. [8 points] Let \( W \) be the differentiable function given by

\[
W(p) = \begin{cases} 
4 \ln(2) + 4 \ln(-p) & \text{if } p \leq -0.5 \\
2 \sin(4p^2 - 1) & \text{if } -0.5 < p < 0 \\
\frac{\arctan(2p - 1)}{p^2} & \text{if } p \geq 0.
\end{cases}
\]

(a) [4 points] Use the limit definition of the derivative to write an explicit expression for \( W'(3) \).
Your answer should not involve the letter \( W \). Do not evaluate or simplify the limit. Please write your final answer in the answer box provided below.

Answer: \( W'(3) = \lim_{h \to 0} \frac{\arctan(2 \cdot 3 + h) - \arctan(2 \cdot 3 - 1)}{(3 + h)^2 - 9} \)

(b) [4 points] With \( W \) as defined above, consider the function \( g \) defined by

\[
g(t) = \begin{cases} 
ct + k & \text{if } t \leq 0 \\
W(-e^t) & \text{if } t > 0
\end{cases}
\]

for some constants \( c \) and \( k \). Find all values of \( c \) and \( k \) so that \( g(t) \) is differentiable.
Show your work carefully, and leave your answers in exact form.
If no such values of \( c \) and/or \( k \) exist, write NONE in the appropriate answer blank and be sure to justify your reasoning.

Solution: Note that for \( t > 0 \), \( g(t) = 4 \ln(2) + 4t \), so

\[
g'(t) = \begin{cases} 
c & \text{for } t < 0 \\
4 & \text{for } t > 0
\end{cases}
\]

(Alternatively, \( g'(t) = W'(-e^t) \cdot -e^t \).) Since we are told that \( W \) is differentiable, we need only to find values so that \( g \) is differentiable at \( t = 0 \).
In order for \( g \) to be differentiable, we need to find values of \( c \) and \( k \) so that

\[
\lim_{t \to 0^-} g(t) = \lim_{t \to 0^+} g(t) \quad \text{and} \quad \lim_{t \to 0^-} g'(t) = \lim_{t \to 0^+} g'(t).
\]

The first equation is true when \( c \cdot 0 + k = W(-e^0) = W(-1) \). Note that since \(-1 < -0.5 \), we have \( W(-1) = 4 \ln(2) + 4 \ln(-(-1)) = 4 \ln(2) \), so \( k = 4 \ln(2) \).

The second equation is true when \( c = -W'(-1) \). Near \( p = -1 \) we have \( W'(p) = 4 \).
Therefore, we need \( c = 4 \).

Answer: \( c = 4 \) and \( k = 4 \ln(2) \)