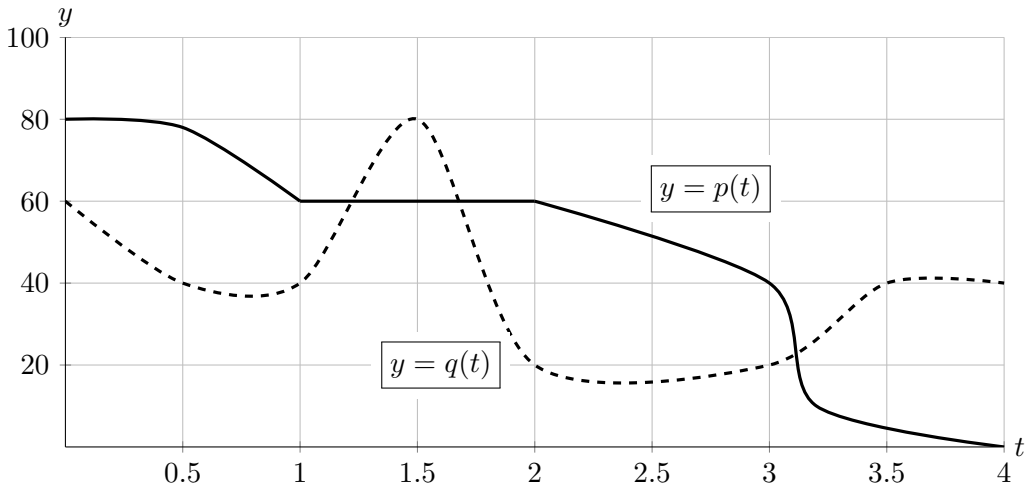
3. [7 points] At the cider mill, Xanthippe makes donuts fastest when she isn't distracted by customers. The rate, in donuts per hour, at which Xanthippe makes donuts t hours after 7 am is modeled by the function $p(t)$. Customers purchase donuts during their visit to the cider mill. The rate, in donuts per hour, at which customers purchase donuts t hours after 7 am is modeled by the function $q(t)$. The graphs of $y = p(t)$ (solid) and $y = q(t)$ (dashed) are shown below. Assume that at 7 am, Xanthippe begins with no donuts in stock.



a. [2 points] At what rate, in donuts per hour, is the number of donuts in stock (donuts produced but not yet sold) increasing/decreasing at 8:30 am? Be sure to circle one of INCREASING or DECREASING.

Solution: At $t = 1.5$, $p(t) - q(t) = -20$. The rate at which donuts are being sold exceeds the rate at which the donuts are being produced at a rate of 20 donuts/hr. Therefore, the number of donuts in stock is decreasing at a rate of 20 donuts/hr.

Answer: INCREASING [DECREASING] at a rate of **20 donuts/hr**

b. [2 points] Write an expression involving p and q for the number of donuts in stock at 10 am. Your answer may involve definite integrals. Do not give approximations.

Solution: $p(t) - q(t)$ is the rate at which the number of donuts in stock is changing t hours after 7 am. By the fundamental theorem of calculus, $\int_0^3 p(t) - q(t)\,dt$ is the change in the number of donuts in stock between 7 am and 10 am. Since there were no donuts in stock at at 7 am, this is the number of donuts in stock at 10 am.

Answer: $\displaystyle\int_0^3 p(t) - q(t)\,dt$

c. [3 points] Xanthippe stops making donuts at 11 am. Assume that after 11 am, customers continue to purchase donuts at a constant rate of 40 donuts per hour until all of Xanthippe's donuts are sold out. Write an expression for the number of hours, starting at 11 am, that it takes for all her donuts to be sold out. Your answer may involve definite integrals. Do not give approximations.

Solution: The number of donuts in stock at 11 am is $\int_0^4 p(t) - q(t)\,dt$. When s hours have passed after 11 am, $40s$ donuts have been sold (assuming all donuts were not already sold), so we want to find s such that $40s = \int_0^4 p(t) - q(t)\,dt$.

Answer: $\displaystyle\frac{1}{40}\int_0^4 p(t) - q(t)\,dt$