6. [4 points] Formulas for a function $g(x)$ and its derivative $g'(x)$ are given below.

$$g(x) = (2 - 4x)e^{-x^2} \quad \text{and} \quad g'(x) = 4(2x+1)(x-1)e^{-x^2}.$$

Find all global extrema of $g(x)$ on the open interval $(0, \infty)$. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all global extrema. Write NONE if appropriate.

Solution: The critical points of g are at $x = \frac{-1}{2}$ and $x = 1$. The first is not in $(0, \infty)$.

Option 1: using a derivative test.

First derivative test: note that e^{-x^2} is always positive. When $0 < x < 1$, $2x + 1$ is positive and $x - 1$ is negative, so $g'(x) < 0$. When $x > 1$, all of the factors in $g'(x)$ are positive, so $g'(x) > 0$. This tells us that g has a local minimum at $x = 1$.

Second derivative test: $g''(x) = -4e^{-x^2}(1-6x-2x^2+4x^3)$, so $g''(1) = -4e^{-1}(1-6-2+4) = 12e^{-1} > 0$, so g has a local minimum at $x = 1$.

After using one of these tests to determine that $g(x)$ has a local minimum at $x = 1$, we need to say something more to answer the question about the location of global extrema.

Since there's only one critical point, and the endpoint $x = 0$ is not included in our interval, the local minimum at $x = 1$ is also the global minimum, and there is no global maximum.

Option 2: using limits

Note that $g(x)$ is a continuous function. We see that $\lim_{x \to 0} g(x) = g(0) = 2$, $g(1) = -2e^{-1}$, and $\lim_{x \to \infty} g(x) = 0$. Since $-2e^{-1} < 0$, we have a global minimum at $x = 1$ and no global maximum.

Answer: global max(es) at $x =$ **None**

global min(s) at $x =$ **1**