7. [5 points] Consider the family of functions given by $g(x) = x\ln(px^2+q)$, for constants p and q. Find values of p and q so that the function has a local extremum at $(1,2)$. Be sure to justify (using calculus) that your resulting function does have a local extremum at $(1,2)$ and to determine the type of extremum. Leave your answers in exact form.
You may find the following information to be useful.

$$g'(x) = \ln\left(px^2+q\right) + \frac{2px^2}{px^2+q} \quad \text{and} \quad g''(x) = \frac{2px(px^2+3q)}{\left(px^2+q\right)^2}$$

Solution: In order to have a local extremum at $(1,2)$, we must have that $g(1) = 2$ and $g'(1) = 0$ or does not exist. So we need

$$1 \cdot \ln(p+q) = 2 \text{ and } \ln(p+q) + \frac{2p}{p+q} = 0.$$

(Note that for the first equation to be true, $p+q > 0$, so $g'(1)$ exists.)
Since $\ln(p+q) = 2$, we have $p+q = e^2$, so the second equation reduces to $2 + \frac{2p}{e^2} = 0$. This is true when $p = -e^2$. Plugging this back into the first equation, we find $q = 2e^2$.
To find whether this is a local min or max, we can plug these values for p and q into the second derivative and evaluate it when $x = 1$. This gives $\frac{-2e^2(-e^2+3\cdot 2e^2)}{e^4}$. The denominator of this is positive, while the numerator is negative, giving us $g''(1) < 0$, so we have a local max at $(1,2)$.

Answer: $p =$ $-e^2$ and $q =$ $2e^2$

Circle one: [LOCAL MAXIMUM] LOCAL MINIMUM