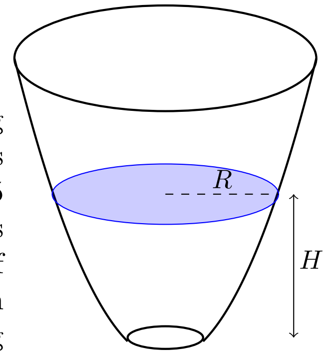
6. [12 points] Water is being poured into a large vase with a circular base. Let $V(t)$ be the volume of water in the vase, in cubic inches, t minutes after the water started being poured into the vase. Let H be the depth of the water in the vase, in inches, and let R be the radius of the surface of the water, in inches.

A formula for V in terms of R and H is given by

$$V = \frac{1}{2}\pi H(R^2 + 8).$$



a. [6 points] Suppose that the water is being poured into the vase at rate of 300 cubic inches per minute. When the depth of the water is 5 inches, the radius of the surface of the water is 4 inches and the radius is increasing at a rate of 1.2 inches per minute. Find the rate at which the depth of the water in the vase is increasing at that time. Show all your work *carefully*.

Answer: ________________________

b. [2 points] Estimate the instantaneous rate of change of H when $t = 3$ if

t	1.5	2.3	3.0	3.2
H	1.3	1.7	1.9	1.95

Show your work and include units.

Answer: ________________________

The problem continues on the next page

c. [4 points] Recall that R gives the radius of the surface of the water, in inches, t minutes after the water started being poured into the vase. Suppose that R is given by $R = m(t)$ and $m'(3) = 0.7$. Use these facts to complete the following sentence:

After the water has been poured into the vase for three minutes, over the next ten seconds, the radius of the surface of the water ...

7. [7 points] Let A and B be positive constants and $f(x) = \dfrac{A(x^2 - B)}{\sqrt{x-3}}$, for $x > 3$. Note that

$$f'(x) = \frac{A(3x^2 - 12x + B)}{2(x-3)^{\frac{3}{2}}} \qquad \text{and} \qquad f''(x) = \frac{3A(x^2 - 8x + 24 - B)}{4(x-3)^{\frac{5}{2}}}.$$

Find all values of A and B so that $f(x)$ has an inflection point at $(8, 2)$. Use calculus to justify that the point $(8, 2)$ is an inflection point. If there are no such values, write NONE.

$A =$ ________________ $B =$ ________________