6. [12 points] Water is being poured into a large vase with a circular base. Let $V(t)$ be the volume of water in the vase, in cubic inches, $t$ minutes after the water started being poured into the vase. Let $H$ be the depth of the water in the vase, in inches, and let $R$ be the radius of the surface of the water, in inches.

A formula for $V$ in terms of $R$ and $H$ is given by

$$V = \frac{1}{2} \pi H (R^2 + 8).$$

a. [6 points] Suppose that the water is being poured into the vase at rate of 300 cubic inches per minute. When the depth of the water is 5 inches, the radius of the surface of the water is 4 inches and the radius is increasing at a rate of 1.2 inches per minute. Find the rate at which the depth of the water in the vase is increasing at that time. Show all your work carefully.

Answer: 

b. [2 points] Estimate the instantaneous rate of change of $H$ when $t = 3$ if

<table>
<thead>
<tr>
<th>$t$</th>
<th>1.5</th>
<th>2.3</th>
<th>3.0</th>
<th>3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>1.3</td>
<td>1.7</td>
<td>1.9</td>
<td>1.95</td>
</tr>
</tbody>
</table>

Show your work and include units.

Answer: 

The problem continues on the next page.
c. [4 points] Recall that $R$ gives the radius of the surface of the water, in inches, $t$ minutes after the water started being poured into the vase. Suppose that $R$ is given by $R = m(t)$ and $m'(3) = 0.7$. Use these facts to complete the following sentence:

*After the water has been poured into the vase for three minutes, over the next ten seconds, the radius of the surface of the water . . .

7. [7 points] Let $A$ and $B$ be positive constants and $f(x) = \frac{A(x^2 - B)}{\sqrt{x - 3}}$, for $x > 3$. Note that

$$f'(x) = \frac{A(3x^2 - 12x + B)}{2(x - 3)^{3/2}} \quad \text{and} \quad f''(x) = \frac{3A(x^2 - 8x + 24 - B)}{4(x - 3)^{5/2}}.$$ 

Find all values of $A$ and $B$ so that $f(x)$ has an inflection point at $(8, 2)$. Use calculus to justify that the point $(8, 2)$ is an inflection point. If there are no such values, write NONE.

$A = \quad \quad \quad \quad B = \quad \quad \quad \quad$