2. [10 points] Jane has a company that produces a protein powder for an energy shake. The cost, in dollars, of producing m pounds of protein powder is given by the function

$$C(m) = \begin{cases} \frac{1}{4}(m+2)^2 + 8 & 0 \le m < 16 \\ 2m + 57 & 16 \le m \le 30. \end{cases}$$

The revenue, in dollars, of selling m pounds of protein powder is given by $R(m) = 5m$.

a. [1 point] What is the price, in dollars, at which Jane sells each pound of the protein powder?

Solution:

Answer= 5

b. [1 point] What is the fixed cost, in dollars, of producing Jane's protein powder?

Solution: Fixed cost is $C(0) = \frac{1}{4}(0+2)^2 + 8 = 9$.

Answer= 9

c. [2 points] Find all values of $16 \le m \le 30$ for which Jane's profit is positive.

Solution: Jane breaks even when $R(m) = C(m)$. That occurs on $16 \le m \le 30$ when $2m + 57 = 5m$. Then Jane breaks even if $m = 19$. We can see that the profit of selling 20 pounds ($m = 20$) is $5(20) - (2(20) + 57) = 3 > 0$. Since both $R(m)$ and $C(m)$ are continuous on $[16, 30]$ then Jane's profit is positive for $19 < m \le 30$.

Answer: $19 < m \le 30$

d. [2 points] Find all the values of $0 \le m \le 30$ where the marginal cost is equal to the marginal revenue for the protein powder. Show all your work to justify your answer.

Solution: Note that

$$MC(m) = \begin{cases} \frac{1}{2}(m+2) & 0 < m < 16 \\ 2 & 16 < m < 30. \end{cases}$$

and $MR(m) = 5$. Hence $MC = MR$ if $\frac{1}{2}(m+2) = 5$ on $0 < m < 16$. Solving for m we get $m = 8$. We do not consider the interval $16 < m < 30$ since in this case, there is no m that yields $MC = MR$.

Answer= 8

e. [4 points] What is the maximum profit that Jane can make if she sells at most 30 pounds of protein powder? Use calculus to find and justify your answer, and make sure to provide enough evidence to fully justify your answer.

Solution: To find the global maximum of the profit $P(m)$ in dollars of selling m pounds of protein powder, we first need to find its critical points on $0 \le m \le 30$. Critical points satisfy either $MC = MR$ ($P'(m) = 0$) or $P'(m)$ does not exist. Hence $m = 8$ is a critical point. The value $m = 16$ is a critical point of $P(m) = R(m) - C(m)$ since $C(m)$ is not differentiable at $m = 16$.

m	0	8	16	30
$P(m) = R(m) - C(m)$	-5	7	-9	33

Answer: 33 dollars.