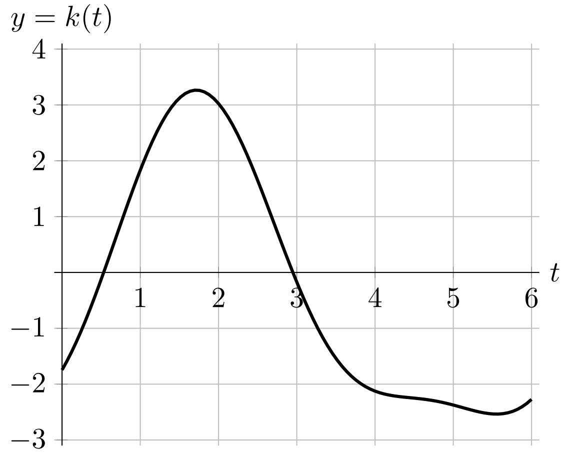
3. [8 points] A group of biologists is studying the population of trout in a lake. Let $k(t)$ be the rate at which the population of trout changes, in thousands of trout per month, t months after the biologists started their study, and let $P(t)$ be the population of trout, in thousands, t months after the study begins. The graph of $y = k(t)$ is shown below for $0 \le t \le 6$.



a. [4 points] Fill in the numbers I. - V. in the blanks below to list the quantities in order from least to greatest.

I. The number zero.

II. $P(4) - P(1)$

III. $\int_1^3 k(t)\, dt$

IV. $\int_3^5 k(t)\, dt$

V. $\int_3^5 k(5)\, dt$

Solution: $V \le IV \le I \le II \le III$

b. [3 points] Suppose $P(2) = 8.6$. Use the graph to find a formula for $L(t)$, the linear approximation for $P(t)$ near $t = 2$.

Solution: Since $k(t) = P'(t)$, then $L(t) = P(2) + k(2)(t-2) = 8.6 + 3(t-2)$

$L(t) = 8.6 + 3(t-2)$

c. [1 point] Use $L(t)$ to approximate the population of trout, in thousands, 1.75 months after the study starts.

Solution: $L(1.75) = 8.6 + 3(1.75 - 2) = 8.6 - 0.75 = 7.85.$

$P(1.75) \approx 7.85$