4. [10 points] Gabe the mouse is swimming alone in a very large puddle of water. He keeps track of his swimming time by logging his velocity at various points in time. Gabe starts at a point on the edge of the puddle and swims in a straight line with increasing speed. A table of Gabe's velocity $V(t)$, in feet per second, t seconds after he begins swimming is given below.

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
$V(t)$	0	0.3	0.4	0.45	0.9	1.2	1.8	2.4	2.7	2.9	3	3.2	3.5

a. [3 points] Give a practical interpretation of the integral $\int_1^{5.5} V(t)\, dt$ in the context of the problem. Be sure to include units.

Solution: The distance Gabe traveled, in feet, in between seconds 1 and 5.5 after he started swimming.

b. [3 points] Estimate $\int_1^{5.5} V(t)\, dt$ by using a right-hand Riemann sum with 3 equal subdivisions. Make sure to write down all terms in your sum.

Solution: If we divide the interval $[1, 5.5]$ in three, we obtain $\Delta t = \dfrac{5.5-1}{3} = 1.5$. Then

$$Right(3) = (V(2.5) + V(4) + V(5.5))\Delta t = (1.2 + 2.7 + 3.2)(1.5) = (7.1)(1.5) = 10.65.$$

Answer=10.65 feet.

c. [1 point] Is your estimate from above an overestimate or an underestimate of the exact value of $\int_1^{5.5} V(t)\, dt$? Circle your answer.

Solution: [✓ OVERESTIMATE] UNDERESTIMATE NOT ENOUGH INFORMATION

d. [3 points] Suppose Gabe wants to use a Riemann sum to calculate how far he traveled between $t = 1$ and $t = 5.5$, accurate to within 0.15 feet. How many times would he have to measure his velocity in this interval in order to achieve this accuracy? Justify your answer.

Solution: Since $V(t)$ is increasing in $[1, 5.5]$ then $|V(5.5) - V(1)|\Delta t \leq 0.15$, where Δt is the possible width of each interval in order for the estimate to be true. Hence $\Delta t \leq \frac{0.15}{2.8}$. Then if N is the number of times Gabe has to measure his velocity to attain its desired accuracy, then $N = \dfrac{5.5-1}{\Delta t} \geq \dfrac{4.5}{\frac{0.15}{2.8}} = 3(28) = 84$

Answer= At least 84 times.