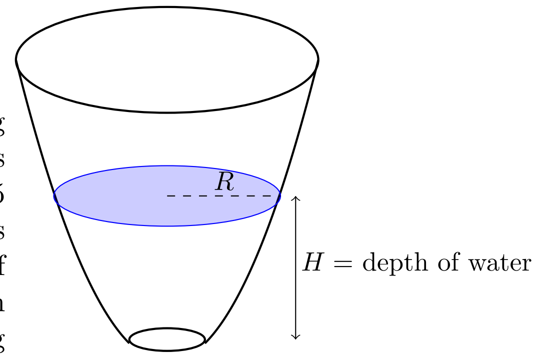
6. [12 points] Water is being poured into a large vase with a circular base. Let $V(t)$ be the volume of water in the vase, in cubic inches, t minutes after the water started being poured into the vase. Let H be the depth of the water in the vase, in inches, and let R be the radius of the surface of the water, in inches.

A formula for V in terms of R and H is given by

$$V = \frac{1}{2}\pi H(R^2 + 8).$$



a. [6 points] Suppose that the water is being poured into the vase at rate of 300 cubic inches per minute. When the depth of the water is 5 inches, the radius of the surface of the water is 4 inches and the radius is increasing at a rate of 1.2 inches per minute. Find the rate at which the depth of the water in the vase is increasing at that time. Show all your work *carefully*.

Solution: Differentiating with respect to time

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{1}{2}\pi H(R^2+8)\right)$$
$$\frac{dV}{dt} = \frac{1}{2}\pi\left(\frac{dH}{dt}(R^2+8) + H\frac{d}{dt}(R^2+8)\right)$$
$$\frac{dV}{dt} = \frac{1}{2}\pi\left(\frac{dH}{dt}(R^2+8) + 2HR\frac{dR}{dt}\right)$$
$$300 = \frac{1}{2}\pi\left(\frac{dH}{dt}((4)^2+8) + 2(5)(4)(1.2)\right) \qquad 300 = \frac{1}{2}\pi\left(24\frac{dH}{dt} + 48\right)$$
$$\frac{dH}{dt} = \frac{\frac{600}{\pi} - 48}{24} \approx 5.96.$$

b. [2 points] Estimate the instantaneous rate of change of H when $t = 3$ if

t	1.5	2.3	3.0	3.2
H	1.3	1.7	1.9	1.95

Show your work and include units.

Solution: $H'(3) \approx \frac{1.95-1.9}{3.2-3} = \frac{0.05}{0.2} = 0.25$ inches per minute.

c. [4 points] Recall that R gives the radius of the surface of the water, in inches, t minutes after the water started being poured into the vase. Suppose that R is given by $R = m(t)$ and $m'(3) = 0.7$. Use these facts to complete the following sentence:

Solution: After the water has been poured into the vase for three minutes, over the next ten seconds, the radius of the surface of the water increases approximately by $\frac{7}{60}$ inches.