9. [9 points] For the following problems, choose the correct answer. If none of the choices are correct, circle NONE OF THESE.

a. [2 points] Which of the following is an antiderivative of the function $1/x + \cos(x)$ for $x > 0$? Circle *all* correct answers.

Solution:

i. $-\frac{1}{x^2} - \sin(x)$

ii. $\boxed{\ln(5x) + \sin(x)}$

iii. $\boxed{\ln(x) + \sin(x) - 20}$

iv. $\ln\left(\frac{1}{x}\cos(x)\right)$

v. $\frac{1}{x^2} + \sin(x)$

vi. NONE OF THESE

b. [2 points] Suppose $f(x)$ is a differentiable, invertible function defined on $(-\infty, \infty)$ with $f'(x) > 0$ for all x. Suppose that $f(3) = 5$ and $f'(3) = 2$. Which of the following statements must be true? Circle *all* correct answers.

Solution:

i. $\boxed{f'(f^{-1}(x)) = \frac{1}{(f^{-1})'(x)}}$

ii. $f'(x)$ is invertible

iii. $(f^{-1})'(x) = \frac{1}{f'(x)}$

iv. $\boxed{(f^{-1})'(5) = \frac{1}{2}}$

v. $f'(2) = \frac{1}{5}$

vi. NONE OF THESE

c. [2 points] If $p(t)$ is an even function that is differentiable on $(-\infty, \infty)$, which of the following must be true? Circle *all* correct answers.

Solution:

i. $\boxed{\int_1^4 p(t)\,dt = \int_{-4}^{-1} p(t)\,dt}$

ii. $\int_{-4}^{4} p(t)\,dt = 0.$

iii. Any antiderivative of $p(t)$ is an even function

iv. $\int_6^8 p(t+3)\,dt = \int_3^5 p(t)\,dt$

v. $\boxed{\int_{-5}^{5} p'(t)\,dt = 0}$

vi. NONE OF THESE

d. [3 points] Suppose the limit definition of the derivative gives

$$g'(-1) = \lim_{h\to 0} \frac{2^{c(-1+h)} + a(-1+h)^3 - (2^{-c} - a)}{h},$$

where a and c are nonzero constants. Which of the following could be the formula for $g(x)$? Circle the *one* best answer.

Solution:

i. $g(x) = 2^{-cx} + ax$

ii. $g(x) = a(x-1)^3 + c^x$

iii. $g(x) = 2^c - a$

iv. $g(x) = 2^{c(x+h)} + ah^3$

v. $\boxed{g(x) = 2^{cx} + ax^3}$

vi. NONE OF THESE