

5. [7 points] Consider the family of functions  $f(x) = bx^5e^{cx}$  with parameters  $b$  and  $c$ . Note that

$$f'(x) = bx^4e^{cx}(cx + 5) \quad \text{and} \quad f''(x) = bx^3e^{cx}(c^2x^2 + 10cx + 20)$$

- a. [2 points] Find all values of  $b$  and  $c$  that make

$$\lim_{x \rightarrow \infty} f(x) = \infty \quad \text{AND} \quad \lim_{x \rightarrow -\infty} f(x) = 0.$$

Conditions for  $b$ : \_\_\_\_\_ Conditions for  $c$ : \_\_\_\_\_

- b. [5 points] Suppose  $b > 0$  and  $c > 0$ . Find the critical point(s) of  $f(x)$  and the  $x$ -coordinates of the local extrema of  $f(x)$ . Your answer must be in exact form and may be expressed in terms of the constants  $b$  and  $c$ . You should use calculus to find and justify your answers. For each answer blank below, write NONE if appropriate.

Critical point(s)  $x =$  \_\_\_\_\_

Local max(es)  $x =$  \_\_\_\_\_ Local min(s)  $x =$  \_\_\_\_\_