5. [7 points] Consider the family of functions \( f(x) = bx^5e^{cx} \) with parameters \( b \) and \( c \). Note that

\[
f'(x) = bx^4e^{cx}(cx + 5) \quad \text{and} \quad f''(x) = bx^3e^{cx}(c^2x^2 + 10cx + 20)
\]

a. [2 points] Find all values of \( b \) and \( c \) that make

\[
\lim_{x \to \infty} f(x) = \infty \quad \text{AND} \quad \lim_{x \to -\infty} f(x) = 0.
\]

Conditions for \( b \): ___________________________  Conditions for \( c \): ___________________________

b. [5 points] Suppose \( b > 0 \) and \( c > 0 \). Find the critical point(s) of \( f(x) \) and the \( x \)-coordinates of the local extrema of \( f(x) \). Your answer must be in exact form and may be expressed in terms of the constants \( b \) and \( c \). You should use calculus to find and justify your answers. For each answer blank below, write NONE if appropriate.

Critical point(s) \( x = \) ___________________________

Local max(es) \( x = \) ___________________________  Local min(s) \( x = \) ___________________________