7. [11 points] Zavier the zoo-keeper is breeding fish in a large aquarium. As the population of fish increases, he notices that the amount of waste in the bottom of aquarium is also increasing. Each week he measures how much waste has accumulated at the bottom of the aquarium. The function $W(t)$ models the amount of waste, in millimeters, at the bottom of the aquarium t weeks after Zavier began his measurements. Note the following information about the function $W(t)$.

- $W(t)$ is continuous on the interval $[0, 9]$.
- During the first 3 weeks, the amount of waste increases exponentially from 102.4 mm at time $t = 0$ to 200 mm at time $t = 3$.
- After 3 weeks, Zavier buys several catfish to eat the waste at the bottom of the aquarium. Over the next 6 weeks, the **rate of change** in the amount of waste (in millimeters per week) is given by the function $g(t) = t^2 - 12t + 26$.

a. [8 points] Write a piecewise defined formula for the **continuous** function $W(t)$ on the interval $[0, 9]$. Show all your work.

$$W(t) = \begin{cases} \underline{\qquad\qquad\qquad\qquad\qquad\qquad} & \text{if } \underline{\qquad\qquad\qquad} \\ \\ \underline{\qquad\qquad\qquad\qquad\qquad\qquad} & \text{if } \underline{\qquad\qquad\qquad} \end{cases}$$

b. [3 points] Zavier continues recording data and notices that, after 50 weeks, the amount of waste is modeled by the function

$$Q(t) = \frac{4(a-t)^2(bt-2)^2}{(3t^2+7t+10)(4-t)(-2t+c)}$$

where a, b, and c are positive constants. What happens to the amount of waste in the long run? Circle the correct answer and fill in the blank if necessary. Your answer may include the constants a, b or c.

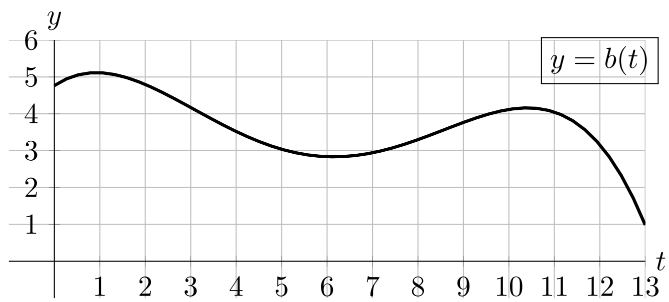
i) It increases without limit.

ii) It approaches zero.

iii) It approaches a positive limit with value L where $L =$ ________________

iv) None of these.

8. [7 points] Ben buys cabbage for his juice business. Let $b(t)$ be the rate at which Ben buys cabbage, in pounds per month, for his business t months after the beginning of 2015. The graph of $b(t)$ is shown below.



a. [3 points] Ben already has 100 lbs of cabbage at the beginning of 2015. Write a mathematical expression involving the function b, its derivative and/or a definite integral that represents the total number of pounds of cabbage Ben bought by the end of 2015.

Answer: ________________________________

b. [2 points] Let $A(t)$ be the amount of cabbage, in pounds, Ben has bought during the first t months of 2015. Suppose $A(5) = 120$. Find a formula for the tangent line approximation $L(t)$ of $A(t)$ near $t = 5$.

$L(t) =$ ________________________________

c. [2 points] Which of the following must be true? Circle your answer.

$L(4.5) < A(4.5)$ $L(4.5) > A(4.5)$ $L(4.5) = A(4.5)$ NOT ENOUGH INFORMATION