1. [10 points] Brianna is riding her unicycle on William Street. As she rides, she passes the Ann Arbor District Library. The function $u(t)$ represents Brianna's location (in meters west of the library) when she has been riding her unicycle for t seconds. The table below shows some values of $u'(t)$, the **derivative** of $u(t)$.

t	0	2	5	10	15	18	20	23	25	30
$u'(t)$	0	1	2	2.5	1.5	0	-1	-1.5	-2	-3

Note the following:

i) $u(23) = 2$.

ii) $u'(t)$ is continuous.

iii) $u'(t)$ satisfies:

- $u'(t)$ is increasing on $(0, 10)$.
- $u'(t)$ is decreasing on $(10, 30)$.

a. [2 points] Circle all of the following intervals on which $u(t)$ could be invertible.

Solution:

[3,8] [2,15] $[5, 20]$ $[10, 25]$ NONE OF THESE

b. [3 points] $u(t)$ is invertible on the interval $[20, 30]$. Let $f(t)$ be the inverse of $u(t)$ on that interval. Calculate $f'(2)$ and include units.

Solution:

$$f'(2) = \frac{1}{u'(u^{-1}(2))} = \frac{1}{u'(23)} = \frac{1}{-1.5} = -\frac{2}{3}.$$

Answer: $-\frac{2}{3}$ seconds per meter.

c. [2 points] Find the value of $\lim_{x \to 23} \frac{u(x) - u(23)}{x - 23}$. If the limit does not exist, write DNE. If it cannot be determined based on the information given, write NI.

Solution:

Answer: $u'(23) = -1.5$.

d. [1 point] Estimate the value of $u''(24)$.

Solution:

$$u''(24) \approx \frac{-2 + 1.5}{2} = -0.25.$$

Answer: -0.25.

e. [2 points] Which of the following values of t could be inflection points of $u(t)$?

Solution:

5 10 17 18 23 NONE OF THESE