

2. [8 points] This problem is based on Brianna's ride with her unicycle. The statement of the previous problem is included for your convenience.

Brianna is riding her unicycle on William Street. As she rides, she passes the Ann Arbor District Library. The function $u(t)$ represents Brianna's location (in meters west of the library) when she has been riding her unicycle for t seconds.

The table below shows some values of $u'(t)$, the **derivative** of $u(t)$.

t	0	2	5	10	15	18	20	23	25	30
$u'(t)$	0	1	2	2.5	1.5	0	-1	-1.5	-2	-3

Note the following:

- i) $u(23) = 2$.
- ii) $u'(t)$ is continuous.
- iii) $u'(t)$ satisfies:
 - $u'(t)$ is increasing on $(0, 10)$.
 - $u'(t)$ is decreasing on $(10, 30)$.
- a. [4 points] Use a right Riemann sum with 4 subintervals of equal size to estimate Brianna's displacement between times $t = 10$ and $t = 30$. Write all the terms in your sum. Include units.

Solution:

Right Riemann sum = $5(u'(15) + u'(20) + u'(25) + u'(30)) = 5(1.5 - 1 - 2 - 3) = -22.5$ meters.

- b. [1 point] Is your answer in part **a** an overestimate or an underestimate? Circle your answer. If there is not enough information circle NI.

Solution:

OVERESTIMATE

UNDERESTIMATE

NEITHER

NI

- c. [3 points] Which of the following *must* be equal to Brianna's average velocity during the time interval $[15, 20]$? Circle all correct answers.

Solution:

- A. $\frac{u(20) - u(15)}{20 - 15}$
- B. $\frac{u'(15) + u'(20)}{2}$
- C. $\frac{u'(15) + u'(18) + u'(20)}{3}$
- D. $\frac{1}{20 - 15} \int_{15}^{20} u(t) dt$
- E. $\frac{1}{20 - 15} \int_{15}^{20} u'(t) dt$

F. $\frac{1}{18 - 15} \int_{15}^{18} u'(t) dt + \frac{1}{20 - 18} \int_{18}^{20} u'(t) dt$

G. $\frac{1}{20 - 15} \left(\int_0^{20} u'(t) dt - \int_0^{15} u'(t) dt \right)$

H. $\frac{1}{20 - 0} \int_0^{20} u'(t) dt - \frac{1}{15 - 0} \int_0^{15} u'(t) dt$

I. NONE OF THESE