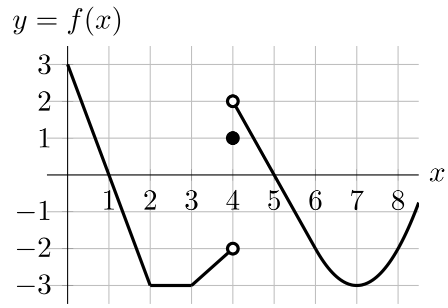
3. [14 points] Suppose $f(x)$ is an even function. A piece of the graph of $f(x)$ is given below. Note that $f(x)$ is piecewise linear for $0 \le x \le 6$. Find the following quantities. If any of their values do not exist, write DNE. If there is not enough information to answer, write NI.



a. [1 point] Find $\lim_{p\to 4^+} f(p)$.

Solution:

Answer: 2

b. [2 points] Find $\lim_{m\to 0} \frac{f(1+m)-f(1)}{m}$.

Solution:

Answer: -3

c. [3 points] Let $g(x) = \frac{1}{\sqrt{4+f(2x)}}$. Find $g'(2.5)$.

Solution:

$$g'(x) = -\frac{1}{2}(4+f(2x))^{-\frac{3}{2}}(2f'(2x)) = -\frac{f'(2x)}{(4+f(2x))^{\frac{3}{2}}} \qquad g'(2.5) = -\frac{f'(5)}{(4+f(5))^{\frac{3}{2}}} = -\frac{(-2)}{4^{\frac{3}{2}}} = \frac{1}{4}$$

Answer: $\frac{1}{4}$

d. [3 points] Recall that $f(x)$ is even. Find $\int_{-3}^{1} (5f(t)-3)\,dt$.

Solution:

$$\int_{-3}^{1} (5f(t)-3)\,dt = 5\left(\int_{-3}^{-1} f(t))\,dt + \int_{-1}^{1} f(t))\,dt\right) - \int_{-3}^{1} 3dt$$

$$= 5(-4.5+3) - 12 = -7.5 - 12 = -19.5.$$

Answer: -19.5

e. [3 points] Let $j(x)$ be an antiderivative of $f(x)$ with $j(5) = 3$. Suppose that $p(x)$ is the quadratic approximation of $j(x)$ near $x = 5$. Find a formula for $p(x)$.

Solution: We know that $p(x) = j(5) + j'(5)(x-5) + \frac{j''(5)}{2}(x-5)^2$. Since $j(x)$ be an antiderivative of $f(x)$ then $j'(5) = f(5) = 0$ and $j''(5) = f'(5) = -2$. Hence

Answer: $p(x) = 3 - (x-5)^2$.

f. [2 points] Find all the values of a with $-3 \le a \le 3$ such that $\int_{-2}^{a} f(x)\,dx = 0$.

Solution:

Answer: $a = -2, 0, 2$