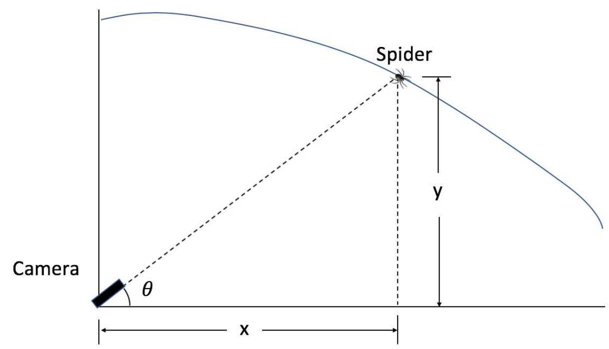
4. [7 points] Casey is making a documentary about the wildlife that lives in a local cave. She found a spider of a new species climbing down along the ceiling of the cave (as shown in the diagram below). Here

- x is the spider's distance to the right, in ft, of the camera
- y is the height, in ft, of the spider from the ground
- θ is the angle, in radians, made by the ground and the line joining Casey's camera and the spider.

The camera is following the spider as it walks along the ceiling of the cave. **Find the rate at which the angle θ is changing** when the following conditions hold:

- The spider is 10 ft above the ground.
- The spider's distance to the right of the camera is increasing at 0.4 feet per second.
- The spider's height is decreasing at a rate of 0.2 feet per second.
- The angle $\theta = \dfrac{\pi}{6}$ radians.



Use the equation

$$\tan(\theta) = \frac{y}{x}.$$

satisfied by the variables x, y and θ to find your answer. Include units. Show all your work.

Solution: Taking derivatives with respect to time we get

$$\frac{1}{\cos^2(\theta)}\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}.$$

Solving for $\frac{d\theta}{dt}$ we get

$$\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}\cos^2(\theta)$$

We are given $y = 10$, $\frac{dx}{dt} = 0.4$, $\frac{dy}{dt} = -0.2$ and $\theta = \frac{\pi}{6}$ then $\tan(\theta) = \frac{y}{x}$ yields $\frac{10}{x} = \frac{1}{\sqrt{3}}$ or $x = 10\sqrt{3}$. Using these values into this equation, we get

$$\frac{d\theta}{dt} = \frac{10\sqrt{3}(-0.2) - 10(0.4)}{(10\sqrt{3})^2}\cos^2\left(\frac{\pi}{6}\right) = \frac{-2\sqrt{3} - 4}{300}\left(\frac{3}{4}\right) = \frac{-\sqrt{3} - 2}{200}$$

Answer: $\dfrac{-\sqrt{3} - 2}{200}$ radians per second.