

1. [9 points] The function  $f(x)$  is invertible and twice differentiable for all real numbers. The table to the right gives several values of  $f'(x)$ , **the derivative of  $f(x)$** . You do not need to show work in this problem, but limited partial credit may be awarded for work shown.

$x$	-2	0	2	3	6
$f'(x)$	6	4	3	0	2

- a. Compute each of the following values **exactly**. If there is not enough information, write NEI. If the value does not exist, write DNE.

i. [2 points]  $\lim_{k \rightarrow 0} \frac{f(-2+k) - f(-2)}{k}$

Answer: 6

- ii. [2 points] Let  $h(x) = 3 \cos(x)f(x)$ . Find  $h'(0)$ .

*Solution:*  $h'(x) = 3 \cos(x)f'(x) - 3 \sin(x)f(x)$

Answer: 12

- iii. [2 points] Let  $g(x) = f\left(\frac{6}{x}\right)$ . Find  $g'(3)$ .

*Solution:*  $g'(x) = f'\left(\frac{6}{x}\right)\left(\frac{-6}{x^2}\right)$

Answer: -2

- b. [1 point] Use the table to give the best possible estimate of  $f''(1)$ .

Answer:  $f''(1) \approx \frac{3-4}{2-0} = \frac{-1}{2}$

- c. [2 points] Suppose that  $f(6) = 0$ . Write a formula for the linear approximation  $L(x)$  of  $f^{-1}(x)$ , **the inverse of  $f(x)$** , at  $x = 0$ .

*Solution:* The slope of  $L(x)$  is  $(f^{-1})'(0) = \frac{1}{f'(6)} = \frac{1}{2}$ .

Answer:  $L(x) = 6 + \frac{1}{2}x$

2. [6 points] Let  $P(h)$  be the current pressure, in millibars (mb), of the air above Ann Arbor at a height of  $h$  meters (m) above the ground.

Use a complete sentence to write a practical interpretation of each of the following equations.

a. [3 points]  $P'(6000) = -0.05$

*Solution:* The pressure at a height of 6100 meters above the ground is about 5 millibars lower than the pressure at 6000 meters above the ground.

b. [3 points]  $\int_0^{4000} P'(h) dh = -510$

*Solution:* The pressure at a height of 4000 meters above the ground is 510 millibars lower than the pressure at ground level.